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UNITY AND DECIMAL
System of Calculating,
-BY-
J. A. HENDERSON, A. M.

INTELLECTUAL AND PRACTICAL

Lightning Calculator.

$$\frac{1}{1}$$

THE UNITY AND DECIMAL SYSTEM.

— BY —

J. A. HENDERSON, A. M.

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PREFACE.

IT is better to know everything about something, than something about everything. Early ideas are not usually true, but need revising and revising before they are thoroughly practical.

There is only one right, but many wrongs, only one right answer, and one right way to find it. Since thought kindles at the fire of thought I have no apology to make for giving to the world the true science of number, and how to apply it easily and accurately, for much time is saved, and above all the true culture and discipline of mind afforded here are above price. The alphabet is the important feature in every science, hence it ought to have our first and best thought, for begin right always right, begin wrong stumble along. Ten is the base of our system of notation. Now let b represent the base of any system of notation, and b divided by b equals the zero power of b. Now to eliminate the literal terms of dividend and divisor, we divide both terms of dividend and divisor, $\frac{b}{b}$, by b, and get the numerical equivalent $\frac{1}{1}$, one one, hence according to axiom first the zero power of the base of any system of numerical notation must be $\frac{1}{1}$. Hence number you observe has two terms which I denominate the unity and unit terms of number the unity term below and the unit term above the line that divides them. Multiplying the unit term by two you get the second character in the alphabet, by three the third &c., and naught the last. The

unity united to the unit contains the idea of addition hence the sign, +, called plus, taking away the unit term and unity term we have the negative sign, —, called minus, the two fundamental signs of arithmetic, the decimal point, ., indicates the omission of the unity term of number. The dividing line is also the sign of division, and $\frac{1}{1}=1$, read one divided by one equals one. The rules of this book are founded on this triune expression, $\frac{1}{1}$, and the whole science of Arithmetic is quickly learned and easily remembered because not only the signs and rules but the analysis and reason of rule proceed directly from the unity and unit terms of number.

Inverting any number tells how many times it is contained in unity, hence the reason of multiplying or dividing follows clearly according to the conditions of the question. This is manifest in the alphabet of numbers, for multiplying the unity term of the $\frac{1}{1}$ by any number tells how many times the number is containd in unity which is equivalent to inverting the number.

Hence in an instant the most intricate and important calculations in every line of business are made. I am the author of the only Intellectual and Practical Lightning Calculator ever published, of which ten editions have been sold. In the mean time, unscrupulous persons have presented my rules and used my name to aid in selling spurious publications under various titles.

J. A. HENDERSON, A. M.

Author and Proprietor.

TABLE OF CONTENTS.

MULTIPLICATION TABLE.

The first line of this table is the "Numerical Alphabet." The second is twice the first; the third is three times, and so on.

The method of acquiring the multiplication table is given on pages 7 and 8.

one one, $\frac{1}{1}$,	two ones, $\frac{2}{1}$,	three ones, $\frac{3}{1}$,	four ones, $\frac{4}{1}$,	five-ones, $\frac{5}{1}$,	six ones, $\frac{6}{1}$,	seven ones, $\frac{7}{1}$,	eight ones, $\frac{8}{1}$,	nine ones, $\frac{9}{1}$,	no ones, $\frac{0}{1}$
2	4	6	8	10	12	14	16	18	0
3	6	9	12	15	18	21	24	27	0
4	8	12	16	20	24	28	32	36	0
5	10	15	20	25	30	35	40	45	0
6	12	18	24	30	36	42	48	54	0
7	14	21	28	35	42	49	56	63	0
8	16	24	32	40	48	56	64	72	0
9	18	27	36	45	54	63	72	81	0

FIRST ADDITION TABLE.

2	1 1							
3	1 2	2 1						
4	1 3	2 2	3 1					
5	1 4	2 3	3 2	4 1				
6	1 5	2 4	3 3	4 2	5 1			
7	1 6	2 5	3 4	4 3	5 2	6 1		
8	1 7	2 6	3 5	4 4	5 3	6 2	7 1	
9	1 8	2 7	3 6	4 5	5 4	6 3	7 2	8 1

The first table gives all combinations of 2 figures, which make 2, 3, 4, 5, &c.

In the second table, the small figures are units which are noticeable in each space, and the tens being understood, thus where there is a cipher, it means ten, the 1 signifies one more than ten, 2 two more than ten, 3 three more than ten &c. Now all combinations of two figures that can possibly

SECOND ADDITION TABLE.

1 9^0	2 8^0	3 7^0	4 6^0	5 5^0	6 4^0	7 3^0	8 2^0	9 1^0
2 9^1	3 8^1	4 7^1	5 6^1	6 5^1	7 4^1	8 3^1	9 2^1	
3 9^2	4 8^2	5 7^2	6 6^2	7 5^2	8 4^2	9 3^2		
4 9^3	5 8^3	6 7^3	7 6^3	8 5^3	9 4^3			
5 9^4	6 8^4	7 7^4	8 6^4	9 5^4				
6 9^5	7 8^5	8 7^5	9 6^5					
7 9^6	8 8^6	9 7^6						
8 9^7	9 8^7							
9 9^8								

be made are given; and must be thoroughly learned in order to become expert. If thoroughly committed to memory it is plainly seen that afterwards the accountant, bookkeeper and all those requiring to use addition have only half the work to perform that they have prior to knowing the combinations.

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The arithmetical alphabet, as written and read,

	one one,	two ones,	three ones	four ones,	five ones,	six ones,	seven ones,	eight ones,	nine ones,	
is	$\frac{1}{1}$,	$\frac{2}{1}$,	$\frac{3}{1}$,	$\frac{4}{1}$,	$\frac{5}{1}$,	$\frac{6}{1}$,	$\frac{7}{1}$,	$\frac{8}{1}$,	$\frac{9}{1}$ &	$\frac{0}{1}$.

The second is two times the first; the third three times the first, etc., up to the last. All numbers larger than nine are expressed by combining two or more of these ten letters or figures, and assigning different values to them, according as they occupy different places.

Ten is expressed by combining one and zero, thus, 10; and omitting the unity or denominator for brevity; two and zero combined make twenty, thus, 20; three and zero, thus, 30, etc. A hundred is expressed by combining the one and two zeros, thus, 100; two hundred, thus, 200. Ten ones make a ten; ten tens make a hundred; ten hundred make one thousand; that is, numbers increase from right to left in a tenfold ratio; hence each removal of a figure one place towards the left increases its value ten times.

The different values which the same figures have are called simple and local values. The simple value of a figure is the value which it expresses when it stands alone, or in the right hand place.

The local value of a figure is the increased value which it expresses by having other figures placed on its right.

NUMERATION.

The art of reading numbers when expressed by figures is called numeration, and can be easily acquired from the following table:

Tredecillions.	Duodecillions.	Undecillions.	Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions	Trillions.	Billions.	Millions.	Thousands.	Units.
685	678	398	746	391	872	281	964	358	123	243	795	937	456	144
XV	XIV	XIII	XII	XI	X	IX	VIII	VII	VI	V	IV	III	II	I

Subtract two from the number of any period, the remainder is the name of the period, above thousands period. The same principle works right and left reading whole and fractional numbers instantly.

We have here fifteen periods of three figures each, beginning at the right hand. The *first* period, which is occupied by units, tens, hundreds, is called *units* period; the second is occupied by thousands, tens of thousands, hundreds of thousands, and is called thousands period; and so on, the orders of each successive period being *units*, *tens* and *hundreds*.

The figures in the table are read thus: 685

tredecillions, 678 duodecillions, 398 undecillions, 746 decillions, 391 nonillions, 872 octillions, 281 septillions, 964 sextillions, 358 quintillions, 123 quadrillions, 243 trillions, 795 billions, 937 millions, 456 thousands, 144 units or ones.

To read numbers expressed by figures: Point them off into periods of three figures each, commencing at the right hand; then, beginning at the left hand, read the figures of each period in the same manner as those of the right hand figure are read, and at the end of each period pronounce its name.

The method of acquiring the multiplication table is of great importance, and is represented thus:

$$\frac{1}{1} \quad \frac{2}{1} \quad \frac{3}{1} \quad \frac{4}{1} \quad \frac{5}{1} \quad \frac{6}{1} \quad \frac{7}{1} \quad \frac{8}{1} \quad \frac{9}{1}$$

Forms the first line of the multiplication table, and may be rehearsed thus: 1 times 1 is 1; 2 times 1 is 2; 3 times 1 is 3; 4 times 1 is 4; 5 times 1 is 5; 6 times 1 is 6; 7 times 1 is 7; 8 times 1 is 8; 9 times 1 is 9.

The second line is 2 times the first, thus:

2 times 1 is 2.

2 times 2 is 2 more than 2, or 4.

2 times 3 is 2 more than 4, or 6.

2 times 4 is 2 more than 6, or 8.

2 times 5 is 2 more than 8, or 10.

2 times 6 is 2 more than 10, or 12

2 times 7 is 2 more than 12, or 14.

2 times 8 is 2 less than 18, or 16.
2 times 7 is 2 less than 16, or 14.
2 times 6 is 2 less than 14, or 12.
2 times 5 is 2 less than 12, or 10.
2 times 4 is 2 less than 10, or 8.
2 times 3 is 2 less than 8, or 6.
2 times 2 is 2 less than 6, or 4.

Thus gaining a knowledge of addition and subtraction, and in fixing in the understanding a knowledge of the table.

The third line is three times the first.
The fourth line is four times the first.
The fifth line is five times the first.
The sixth line is six times the first.
The seventh line is seven times the first, etc.

ADDITION.

3
2
5
7
6
4
8
9
3
2
4
6
—
59

Commence at the units column, add two figures at once, omitting the words *and* and *are*, stopping between forty and fifty. Thus: 10, 15, 32, 42, writing the 2 at the right of the 6; begin again—12, 17, 19, writing down the 9; carry 1 for the 19 and 4 for the catch figure, making 59.

46
53
47
98
76
34
62
47
56
—
519

Two or more columns may be added in a similar way

RULE.—*For adding one or more columns, commence at the right hand column; find the sum; add all except the right hand figure to the second column; proceed in like manner with all the remaining columns.*

THE LIGHTNING PROCESS BY COMBINATION.

First four rows are miscellaneous; second four are the complement of the first, taking 9 as the base:

763712367
812367842
176542051
534256729
236287632
187632157
823457948
465743270

4579831022
18

RULE.—*Prefix the number of nines to the odd row, strike a line and subtract the number of nines.*

HENDERSON'S SECOND BUSINESS METHOD OF ADDITION.

788
8476
3155
4567
5763
6789
6345
6789
6789
5765
4678
9367
5678
74161

Rule.—Commence at the right hand column; find the sum, which is 81, write the 8 over the second column, and the one under the first; find the sum of the second column, which is 86, write the 8 over the third column, and the six under the second column, proceeding in this way to the last column, when you write down the whole result.

Note.—You write the 8 over the column of tens because it is tens; when adding, observe to add this figure in the column of tens, etc., for all the other columns. When the sum of a column is one hundred or more, write the right hand figure under the column, the other two figures over the next two columns.

Third Business Method.—Commence at the left hand column, find the sum, which is 67, write down the result; write the sum of the next column under one place to the right, &c.; for the remaining columns; write their sum underneath for the complete result, thus:

67 is the sum of the first left hand column.
 63 is the sum of the second left hand column.
  78 is the sum of the third left hand column.
   81 is the sum of the fourth, or first right hand column.
74161

The fourth business method is similar to the third; commence at the right hand column instead of the left, write down the sum of each column one place to the left instead of to the

right; thus, in the given example, the sum of
the first column is 81
the second column is 78
the third column is 63
the fourth column is 67
74161

The first business method will aid in the application of the other three. The lightning process of combination is only for mental discipline, or exercise in addition and subtraction. and preparing the student for the fundamental proof of the science of number.

The student ought to have a brilliant idea of the simple and local value of figures, to thoroughly understand whole and fractional numbers. You see on page fifth, to omit the character that represents *unity* for brevity in writing whole numbers; but it appears in every fraction divided into a certain number of equal parts and represents the denominator of the fraction. Let us examine the simple and local value of figures before presenting proof of the rule of addition.

Take the number 987654321. Now, the denominator of this number is one, or unity, understood. This number contains all of the characters in the alphabet of numbers except the last character. The first numerical character is the zero power of $\frac{10}{1}$, the base of numbers, which is $\frac{1}{1}$; the second character is two times the zero power of the base, or $\frac{2}{1}$, etc.

Explained in the Appendix, page 111.

Let the small characters 012445, &c., indicate

the zero power of the base of numbers, the first power, second power, third power, &c. Writing the zero power under and a little to the right of the units figure, the one under the tens a little to the right, &c. Thus in the above number $9_8 8_7 7_6 6_5 5_4 4_3 3_2 2_1 1_0$. Now, observe 1_0 is one time the zero power of ten, or 1, omitting the unity; 2_1 is two times the first power of the base, or 20; 3_2 is three times the second power of the base, or 300, &c., for the local valud of other figures.

PROOF OF FUNDAMENTAL RULE OF ARITHMETIC.

I know of no author who has succeeded in demonstrating and presenting satisfactorily this important feature in the science of numbers.

It will become apparent to every mind, by beginning at the right place,——the origin and demonstration of the New Alphabet of Numbers found on page 111 of this book,——where we find the zero power of the base of numbers 10^0 equivalent to the first numerical character, $\frac{1}{1}$. Hence we find that every quantity or number necessarily has one, or unity, to represent the denominator or denomination. When this is thoroughly understood, the denominator may be omitted in writing numbers, but must never be omitted in the explanation of fractions, the method of in-

terest, etc., because nearly every rule is framed upon the Alphabet of Numbers.

PROOF OF ADDITION.

We will take a number, and examine the nature of the local value of the characters composing the number.

Example: $8_3\,7_2\,4_1\,4_0\,.2_{-1}\,4_{-2}\,5_{-3}$. Now, the value of the first 4 at the left of the decimal point, is four times the zero power of 10, the base of numbers, or four ones. The power of the base is indicated by placing the exponent below the figure, a little to the right.

The value of the second figure to the left of the decimal point is found by multiplying it into the first power, thus: four times the first power of $\frac{10}{1}$, the base, is $\frac{40}{1}$, the value of the second figure toward the left.

The value of the third figure toward the left is seven times the second power of the base, or seven hundred.

The value of the fourth figure toward the left is eight times the third power of the base, or eight thousand, etc., the power of the base increasing with each removal toward the left.

Now, commencing at the decimal point, we find the value of the first figure to the

right is produced by multiplying the 2 by ten minus the first power, which is equivalent to the base inverted.

The value of the second figure is found by multiplying 4 by ten minus the second power, or what is equivalent, by the second power of the base inverted.

Thus: the base of numbers is $\frac{10}{1}$, inverted becomes $\frac{1}{10}$, and the second power of $\frac{1}{10}$ is $\frac{1}{100}$, and 4 times $\frac{1}{100}$ equals $\frac{4}{100}$, the value of the second figue to the right of the decimal point.

The value of the third figure to the right is five times ten minus the third power of the base, or simply five times the third power of the base inverted, which equals $\frac{5}{1000}$.

Now, for the proof of Addition by casting out 9's, and detecting errors that may occur by the common method in use. It must be obvious, that should a figure be misplaced, or transposed, the proof would be the same, while the answer would be incorrect. Hence this is not absolute proof of correct work.

Following is a method of absolute proof:

Example:
$$\begin{array}{r} 4_3 5_2 6_1 3_0 \\ 2_3 3_2 4_1 5_0 \\ \hline 6_3 9_2 0_1 8_0 \end{array}$$

$$\left.\begin{array}{ll} \text{Figure } 4_3 = 4(10^3-1)+4 \\ \text{“} \quad 5_2 = 5(10^2-1)+5 \\ \text{“} \quad 6_1 = 6(10^1-1)+6 \\ \text{“} \quad 3_0 = 3(10^0-1)+3 \end{array}\right\} = 4_3 5_2 6_1 3_0$$

SECOND NUMBER.

$$\left.\begin{array}{ll} \text{Figure } 2_3 = 2(10^3-1)+2 \\ \text{“} \quad 3_2 = 3(10^2-1)+3 \\ \text{“} \quad 4_1 = 4(10^1-1)+4 \\ \text{“} \quad 5_0 = 5(10^0-1)+5 \end{array}\right\} \begin{array}{c} = 2_3 3_2 4_1 5_0 \\ \hline 6_3 9_2 0_1 8_0 \end{array}$$

First figure of answer: $6_3 = 6(10^3-1)+6$

Second “ “ $9_2 = 9(10^2-1)+9$

Third “ “ $0_1 = 0(10^1-1)+0$

Fourth “ “ $8_0 = 8(10^0-1)+8$

We observe 10^3-1 contains one hundred and eleven 9's. Four times 111 9's plus 4 equals the value of the first 4 considered. Hence, by casting the 9's out of this 4 we cast out 444 9's, and have 4 remaining.

Cast the 9's out of the next figure in like manner, adding the remainder to the next figure, etc., to the last figure of the example, writing down the remainder, or 0, as the case may be. Proceed in like manner, casting the 9's out of the figures next in order, when, if the work is correct, a remainder will be found equal to that found by casting the 9's out of the numbers separately.

It is evident the 9's in the numbers to be added, must equal the number of 9's in their sum.

Henderson's Universal Law of Multiplication.

Number has two terms, the unity and unit, the unity term corresponding to the multiplier and the unit to the multiplicand. Hence, the law or rule of operation must be manifest in number.

Illustration:—

21,	RULE for two figures by two—1st
21,	term by 1st, 1st by 2d and 2d
441	by 1st, and 2d by 2d.
11	
321	The language of the rule for any
321	three figures by any other three
103041	proceeds from the number 321 by 321: thus

RULE.—1st term by the 1st; 1st by 2d and 2d by 1st, 1st by 3d and 2d by 2d and 3d by 1st, now 2d by 3d and 3d by 2d, and 3d by 3d. N. B.—Changing the figures does not change the terms in any example.

The language of the law of any four figures by any other four figures, proceeds from the number 4321 by 4321, thus:

4321	thus reaching more than millions
4321	and bringing every figure repre-
18671041	senting the product in the right

place while the highest thought in the operation is only twenty-seven.

```
4 5 4 3 2 1
      54321
      54321
 ----------
 2950771041
```

RULE.—1st term by 1st; 1st by 2d and 2d by 1st; 1st by 3d, 2d by 2d and 3d by 1st; 1st by 4th, 2d by 3d, 3d by 2d and 4th by 1st; 1st by 5th, 2d by 4th, 3d by 3d, 4th by 2d and 5th by 1st; now, 5th by 2d, 4th by 3d, 3d by 4th and 2d by 5th; now, 5th by 3d, 4th by 4th and 3d by 5th; now, 5th by 4th and 4th by 5th, and 5th by 5th, &c., for any number of terms.

```
0 1 2 2 2 1
   20341
    3142
--------
63911422
```

You may commence at left or right, bringing the carrying figure below, or you may reverse the multiplier, thus:

```
4 5 4 3 2 1
      54321
      12345
 ----------
 2950771041
```

The application of the rule remains the same, the first term of the multiplier is then at the left.

The fundamental law of multiplication, as presented here, is of utmost importance to the young student and business man; for it is necessary in every page of mathematical science, and all the works and walks of man.

Examples.—

1. What does 28 yards of prints cost, at 7 cents per yard?

```
                          5
Statement and solution:  28
                          7
                       -----
                       $1.96 ans.
```

2. What does 243 yards cost, at 9 cents per yard.

Statement and solution:
32
243
9
$21.87 ans.

3. What does 4578 yards cost, at 6 cents per yard?

Statement and solution:
344
4578
6
$274.68 ans.

4. What does 3457 yards cost, at 8 cents per yard?

Statement and solution:
345
3457
8
$276.56 ans.

5. What does 23 yards cost, at 26 cents per yard?

Statement and solution:
11
23
26
$5.98 ans.

6. What does 4324 yards cost, at 32 cents per yard?

Statement and solution:
111
4324
32
$1383.68 ans.

7. What does 326 yards cost, at $1.13 per yard?

Statement and solution:
111
326
1.13
$368.38 ans.

8. What does 1244 yards cost, at $2.33 per yard?

Statement and solution:

```
     1221
     1244
     2.33
 --------
 $2898.52 ans.
```

9. What does 42 yards cost, at $7.24 per yard?

Statement and solution:

```
     22
   $7.24
      42
 -------
 $304.08 ans.
```

10. What does 87493 yards cost, at $1.31 per yard?

Statement and solution:

```
      33331
      87493
       1.31
 ----------
 $114615.83 ans.
```

11. What does 32433 yards cost, at $21.34 per yard?

Statement and solution:

```
     233321
      32433
      21.34
 ----------
 $692120.22 ans.
```

Read the rule until the law of operation is established clearly and thoroughly in the mind.

For absolute proof see page 184.

LONG DIVISION.

The terms called dividend and divisor correspond to multiplicand and multiplier, or the unit and unity terms of number. A few examples taken from the origin of the rule of multiplication ought to be sufficient to make plain the rule of dividing, for the rule of division must be seen first in the multiplication table, then in the rule of multiplying.

Example 1.—Divide 441 by 21. Write No. 21 at the left of 441, thus 21)441 and quotient figures at the right.

```
21)441(21
    21
   ---
     0
```

Subtracting products mentally without setting them down.

Example 2.—Divide 103041 by 321.

```
321)103041(321
      674
     ----
      321
```

Example 3.—Divide 18671041 by 4321.

```
4321)18671041(4321
      13870
      -----
        9074
        ----
        4321
        ----
           0
```

Example 4.—Divide 295077041 by 54321.

```
54321)2950771041(54321
      234721
      ------
       174370
       ------
        114074
        ------
          54321
          -----
              0
```

You may write the dividend in the unit term of number and the factors of the divisor in the unity term of number and the common factors vanishing, the quotient appears in factors in the unit and unity terms. Thus; divide

$1728\times 8\times 4\times 4$ by 2×128.

Statement. $\dfrac{1728\times 8\times 4\times 4}{2\times 128}=864$ quotient.

You can find quotient by my complement and supplement rule of division found in back part of the book. You can also find quotient by removing the decimal point which always represents the unity term of number or divisor and multiplying. Thus divide

123456.789 by 125
            8

986754.312 quotient or answer.

N. B. You may reverse the divisor same as multiplier in multiplication.

HENDERSON'S DECIMAL METHOD OF COMPUTING INTREST.

The base of our system ot notation being 10, numbers increase and diminish in a tenfold ratio; increasing from the decimal point to the left, and decreasing from the decimal point towards the right. Hence, to divide any number by 10, remove the point one place to the left.

To divide any number by 100, remove the point two places to the left.

To divide any number by 1000, remove the point three places to the left.

To multiply any number by 10, remove the point one place to the right.

To multiply any number by 100, remove the point two places to the right.

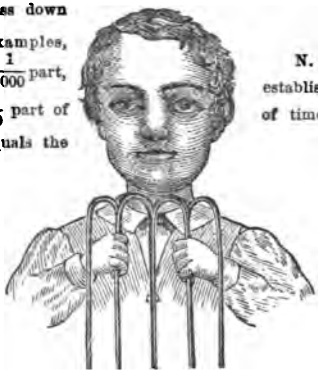
To multiply any number by 1000, remove the point three places to the right.

INTEREST.

Since the interest is generally a part of the principal, the method of calculating it, will come under the method of dividing. The rule establishes the time when a dollar makes a cent, and we remove the point two places to the left; for one hundredth of the principal equals the interest.

IN AN INSTANT THE WORK IS EXECUTED, ON ALL EXAMPLES, AT ANY RATE, FOR FIVE BUSINESS PERIODS OF TIME.

The lines pass down through all examples, and tell when $\frac{1}{1000}$ part, $\frac{1}{100}$ part, and $\frac{1}{10}$ part of the principal equals the interest.



N. B.—The Rule establishes the periods of time.

$87 6 5 4 . 3 2
1 8 7 6 . 4 1

RULE FOR ALL RATES.—*Invert the rate, annex a cipher, and prefix the point.*

Inverting the rate demonstrates the time it takes a dollar to earn a cent, and removing the decimal point two places to the left gives the interest of any sum of money for that time, and rate: annexing a cypher to that time demonstrates the time it takes a dollar to earn ten cents, and removing the point one place gives the interest of any sum of money; prefixing the point demonstrates the time it takes a dollar to earn a mill, and removing the decimal point three places to the left gives the interest of any sum of money for that time; increase or diminish results to suit the required time.

EXAMPLE.—*Rate* 9 *per cent. per annum: inverting the rate we have one-ninth of a year, or* 40 *days, a dollar earns one cent; hence the* $\frac{1}{100}$ *part of the principal is the interest of any sum for* 40 *days, at* 9 *per cent. per annum; and we remove the point two places to the left to find the interest for* 40 *days; now annexing a cypher to forty we have* 400 *days, the time it takes a dollar to earn ten cents; hence* $\frac{1}{10}$ *of the principal equals the interest; and we remove the point one place to find the interest for* 400 *days; prefixing the point, or dividing by ten, we have* 4 *days, or the time it takes a dollar to earn a mill, when* $\frac{1}{1000}$ *part of the principal equals the interest, and we remove the decimal point three places to the left in any example. Thus: per annum,* 9 *per cent., inverting the rate, we have* $\frac{1}{9}$ *of a year, or* 40 *days; annexing a cypher, we have* 400 *days; prefixing the point, or dividing by* 10, *we have* 4 *days; thus the rule establishes the periods of time. Example—* $16,000.00, *at* 9 *per cent. per annum, removing the point one place to the left, we have* $1600.00, *the interest for* 400 *days; removing the point two places to the left, we have* $160.00, *the interest for* 40 *days; removing the decimal point three places, we have* $16.00, *the interest for* 4 *days. On this or any other example, at* 9 *per cent. per annum, all rates are handled absolutely the same.*

By this method a world of work is done in the twinkling of an eye, and the way opened to the answer of every example in interest.

The rate is 2 per cent. per month, 2 inverted is ½, or 15 days, the point removed two places to the left, all examples are calculated for that rate and date, 10 times half a month or five months, the point removed one place to the left, all examples are calculated. One tenth of 15 days, or a day and a half, the point removed three places to the left, all examples are performed for that time and rate.

	1½ days int. at 2 per cent. per month.	15 days int. at 2 per cent. per month.	5 months int. at 2 per cent. per month
$10,	0	0	0.50
25,	6	5	0.00
2,	4	7	5.30
9,	4	6	0.50
	2	5	0.31

Simply increasing or diminishing the results we find the answer for any other time.

CALCULATING INTEREST.

RULE.—*Invert the rate, annex ciphers and prefix points.* The rule establishes the periods of time, at any rate, that a dollar is earning a cent, dime, dollar, mill, tenth of a mill, hundreth of a mill, &c. Also the sum of money that earns a cent, dime, dollar, mill, tenth of a mill, hundreth of a mill, &c., at any rate per cent. Hence you can find the interest of any sum of money at any rate by removing the decimal point in time or money.

10	per cent,	3.6 days,	36 days,	1 year,	10 years.
9	"	4 "	40 "	400 days,	4000 days.
8	"	4.5 "	45 "	15 mos.,	150 mos.
6	"	6 "	2 mos.,	20 "	200 "
5	"	7.2 "	72 days,	2 years,	20 years.
4	"	9 "	3 mos.,	30 mos.,	300 mos.
3	"	12 "	4 "	40 "	400 "
4½	"	8 "	80 days,	800 days,	8000 days.
3⅓	"	10.8 "	108 "	3 years,	30 years.

	4 2/7	pr. ct.,	8.4 ds.,	84 ds.,	2⅓ yrs.,	23⅓ ys.
	8 4/7	"	4.2 "	42 "	14 mos.,	140 mos.
Pr. mo.,	1½	"	2 "	20 "	200 ds.,	2000 ds.
"	2	"	1.5 "	15 "	5 mos.,	50 mos.
"	2½	"	1.2 "	12 "	4 "	40 "
"	1	"	3 "	1 mo.,	10 "	100 "
"	¾	"	4 "	40 ds.,	400 ds.,	4000 ds.
"	1¼	"	2.4 "	24 "	8 mos.,	80 mos.

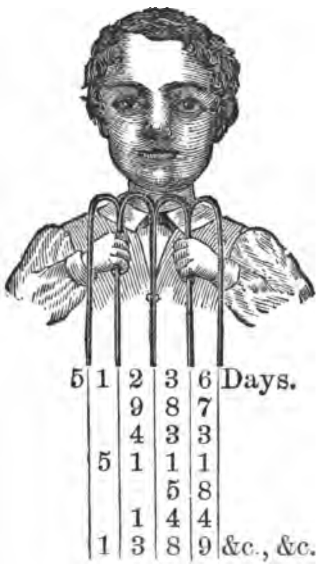
Illustration: $ 8760.75
341.80
96943.78
50500.25
600.00 &c., &c.

All examples are performed without making a figure for the periods of time established by rule.

Per ann.	8 per cent,	$4.50	45.00	450.00	4500.00
"	9 "	4.00	40.00	400.00	4000.00
"	12 "	3.00	30.00	300.00	3000.00
"	6 "	6.00	60.00	600.00	6000.00
"	3 "	12.00	120.00	1200.00	12000.00
"	7½ "	4.80	48.00	480.00	4800.00

Now the interest is found by removing the decimal point on the number of days, and prefixing the sign of dollars, and all periods of time are reached without making a figure.

The lines pass down through all sums of money and all periods of time and represent the decimal point, and you read the interest from money or time.



Rule for working one example at once, and analysis.

RULE.—Take for the unity term of number the time it takes a dollar to earn a mill, cent, dime or dollar, and for the unit term the principal united by × to the time the note is on interest, and the complete analysis stands before you for any example in simple interest; or take for the unity term of number the number of dollars it takes to earn a cent, a dime, a dollar, a mill, a tenth of a mill, &c., and for the unit term the principal united to the given time by ×, and the complete analysis is in the statement.

Give interest of \$480.00 for 93 days @ 6 per cent. Form of statement,

$$\frac{\$480.00 \times 93}{6} = \$7.440.$$

Of \$400.00, 7 months, 15 days @ 6 per cent.

$$\frac{\$400.00 \times 7.5}{2} = \$15.00.$$

Of \$160.00 for 3 years, 7 months, 27 days @ 6 per cent. 3 years, 7 months, 27 days= 43.9 month. $\frac{\$160.00 \times 43.9}{20} = \$35.12.$

Simply unite principal and time by × for unit term of statement, and for unity term the time a dollar is earning a mill, a cent or a dime. Above first rule of interest gives unity term of statement.

METHOD OF SQUARING NUMBERS BY THEIR COMPLEMENT AND SUPPLEMENT.

The complement of a number is the difference between the number and some particular number above it.

The supplement of a number is the difference of a number and some number below it.

$(99)^2 = 9801$. Take the complement of 99 from it, call it hundreds, and add the square of the complement.

$(98)^2 = 9604$. Now 2, the complement of 98 from $98 = 96$; call it hundreds, and add the square of 2, and we have 9604, the square of 98.

$(97)^2 = 9409$. The complement 3 from $97 = 94$; call it hundreds, and add the square of 3, and we have the square of 97.

$(96)^2 = 9216$. The complement of 96 is 4; 4 from $96 = 92$, call it hundreds, and add the square of 4, and we have the square of 96.

$(95)^2 = 9025$. The completent of 95 is 5; $95 - 5 \quad 90$, call it hundreds, and add the square of 5, and we have the square of 95.

$(101)^2 = 10201$. The supplement of 101 is 1;

1 added to 101 is 102, call it hundreds, and add the square of 1, and we have 10201 the square of 101.

$(102)^2 = 10404$. The supplement is 2, added to 102 is 104, call it hundreds, and add the square of 2, and we have 10404 the square of 103.

Rule for squaring whole numbers and fractions. Increase the number by its supplement, multiply it by the base and add the square of the supplement; diminish the number by its complement, multiply it by the base and add the square of the complement.

$(103)^2 = 10609$. The supplement 3 added, call it hundreds, and add the square of 3.

$(104)^2 = 10816$.

$(1001)^2 = 1002001$. The supplement is 1 added to $1001 = 1002$, call it thousands, and add the square of 1, and it equals 1002001. $(1002)^2 = 1004004$. $(1003)^2 = 1006009$. $(1004)^2 = 1008016$.

$(999)^2 = 998001$. The complement is 1 from 999 equals 998, call it thousands, and add the square of 1, and we have the square of the number. $(998)^2 = 996004$. $(997)^2 = 994009$. $(996)^2 = 992016$. $(995)^2 = 990025$. $(994)^2 = 988036$, etc.

Take any number that is easy to multiply by for the base 10, 20, 30, 50, 80, 100, 1000, etc.

$9^2 = 81$. The complement of 9 is 1, 1 from

9 leaves 8, call it tens and add the square of 1, and we have the square of 9.

$8^2 = 64$. The complement of 8 is 2, 2 from 8 leaves 6, call it tens, and add the square of 2, and we have the square of 8.

$(11)^2 = 121$. The supplement of 11 is 1, 1 added to 11 is 12, call it tens and add the square of 1, and we have the square of 11.

$(12)^2 = 144$. The supplement is 2, 2 added to 12 is 14, call it tens and add the square of 2, and we have the square of the number.

$(13)^2 = 169$. The supplement of 13 is 3, 3 added is 16, call it tens and add the square of 3, and we have the square of the number.

$(14)^2 = 196$. $(15)^2 = 225$.

$(19)^2 = 361$. The complement is 1, 1 from 19 leaves 18, 18 multiplied by 20, equals 360, add the square of 1, and we have the square of the number.

$(18)^2 = 324$. $(17)^2 = 289$. $(16)^2 = 256$. $(21)^2 = 441$. $(22)^2 = 484$. $(49)^2 = 2401$. The complement is 1, 1 from 49 is 48, call it fifties, and add the square of 1, and we have 2401, Ans.

$(51)^2 = 2601$. $(52)^2 = 2704$. $(53)^2 = 2809$.

To multiply numbers.

To multiply two numbers, find their mean, square it, and subtruct the square of half their difference.

$19 \times 21 = 399$. The mean is 20, the square of 20 is 400; $400 - 1^2$ is 399, product of $19 \times$

21, 18 × 22. The mean is 20, the square of 20 is 400, 2^2 is 4; 4 from 400 leaves 396, the product, 17×23=391. The square of 3 is 9; 9 from 400 leaves 391 the product.

16×24=384. The square of 4 is 16. 16 from 400 leaves 384 the product.

15×25=375. The square of 5 is 25. 25 from 400 leaves 375 the product.

29×31=899. The mean is 30. The square is 900, minus the square of 1 is 899 their product.

28×32=896. The square of the mean is 900, minus the square of 2 is 896 the product.

27×33=891.	26×34=884.	25×35=875.
39×41=1599.	38×42=1596.	37×43=1591.
36×44=1584.	35×45=1575.	34×46=1564.
49×51=2499	48×52=2496.	47×53=2491.

GREATEST COMMON FACTOR OR DIVISOR

What is the greatest common divisor of 21 and 77. Separating the numbers into their prime factors we have 21=7×3, 77=7×11, hence 7 is the greatest common factor or the greater common divisor of the two numbers.

RULE.—*Separate the numbers into their prime factors. The product of all the factors that are common will be the greatest common divisor.*

What is the greatest divisor of 25 and 60. 25 $=5\times5$, $60=5\times3\times2\times2$? Hence 5 is the greatest common divisor.

What is the greatest common divisor of 5, 15 and 20?

What is the greatest common divisor of 36, 18, 24 and 12. $36=6\times6$, $18=6\times3$, $24=6\times4$, $12=6\times2$? Hence 6 is the greatest common factor or divisor.

What is the greatest common divisor of 135 and 225?

What is the greatest common divisor of 4, 8, 12, 16?

What is the greatest common divisor of 25 and 75?

What is the greatest common divisor of 13 and 65?

What is the greatest common divisor of 14 and 42?

LEAST COMMON MULTIPLE.

A multiple of a number is any number which contains it as a factor.

A common multiple of two or more numbers is any number which contains them all as factors.

The least common multiple of two or more numbers is the least number which contains them all as factors. Hence it follows a multiple

of a number must contain all the prime factors of that number.

A common multiple of two or more numbers must contain all the prime factors of those numbers.

The least common multiple of two or more numbers must be the least number that contains all the prime factors of those numbers.

RULE.—*The product of all the prime factors of that number having the greatest number of prime factors, and those prime factors of the other numbers not found in the factors of the nnmber taken, will be the least common multiple.*

What is the least common multiple of 12 and 18? $12=2\times2\times3$, $18=2\times3\times3$. The least common multiple is $2\times2\times3\times3$ or 36.

What is the least common multiple of 4 and 6?

What is the least common multiple of 18 and 36?

What is the least common multiple of 4, 6, 8 and 10?

What is the least common multiple of 2, 4, 6, 9 and 18?

What is the least common multiple of 2, 3, 4, 5 and 6?

RULE FOR ADDING AND SUBTRACTING FRACTIONS.

First make the fractions similar by reducing them to the same denominator. Add the numer-

ators and place the sum over the common denominator. In subtraction write the difference of the numerators over the common denominator.

What is the sum of $\frac{5}{12}$ and $\frac{1}{4}=\frac{2}{3}$.
What is the sum of $\frac{9}{10}$ and $\frac{4}{7}=$
What is the sum of $\frac{2}{5}$ and $\frac{5}{8}=1\frac{1}{40}$.
What is the sum of $\frac{8}{9}$ and $\frac{1}{6}=1\frac{1}{18}$.
What is the sum of $\frac{3}{8}$ and $\frac{2}{3}=1\frac{1}{24}$.
From $\frac{3}{4}$ subtract $\frac{1}{3}=\frac{5}{12}$.
From $\frac{2}{3}$ subtract $\frac{3}{8}$. $\frac{2}{3}=\frac{16}{24}$, $\frac{3}{8}=\frac{9}{24}$, $\frac{16}{24}-\frac{9}{24}=\frac{7}{24}$.
From $\frac{6}{25}$ take $\frac{1}{5}$. $\frac{1}{5}=\frac{5}{25}$, $\frac{6}{25}-\frac{5}{25}=\frac{1}{25}$.
What is the sum of $3\frac{1}{2}$, $2\frac{1}{3}$, $4\frac{1}{2}$, $5\frac{1}{2}=15\frac{5}{6}$.
Add the fractions and whole numbers separately.
What is the sum of $9\frac{1}{3}$, $6\frac{1}{2}$, $7\frac{2}{3}=23\frac{1}{2}$.
From $8\frac{1}{2}$ take $3\frac{1}{4}$, $\frac{1}{2}=\frac{2}{4}$, $\frac{2}{4}-\frac{1}{4}=\frac{1}{4}$. $8-3=5$; $5+\frac{1}{4}=5\frac{1}{4}$.
From $23\frac{2}{3}$ take $9\frac{1}{2}$. $\frac{2}{3}=\frac{4}{6}$, $\frac{1}{2}=\frac{3}{6}$, $\frac{4}{6}-\frac{3}{6}=\frac{1}{6}$, $25-9=14+\frac{1}{6}=14\frac{1}{6}$.

GENERAL PRINCIPLES OF FRACTIONS.

Multiplying the numerator multiplies the fraction.

Dividing the numerator divides the fraction.

Multiplying the denominator divides the fraction.

Dividing the denominator multiplies the fraction.

Multiplying both terms of the fraction by the same number does not change its value.

Fractions are called similar when they have a common denominator, as $\frac{3}{8}$, $\frac{7}{8}$, $\frac{5}{8}$, $\frac{1}{8}$.

Dissimilar fractions are fractions which are not alike, as $\frac{2}{7}$, $\frac{2}{4}$, $\frac{3}{5}$, ^{5}, $\frac{4}{9}$.

The numerators of similar fractions only can be added.

The common denominator is written under the sum or difference.

Multiply $\frac{4}{11}$ by $8=\frac{32}{11}=2\frac{10}{11}$.

Multiply $\frac{5}{14}$ by $14=\frac{70}{14}=5$.

Multiply 40 by $\frac{5}{8}=5\times5=25$.

Multiply $3\frac{1}{2}$ by 6. Multiply the whole number and fraction separately. $6\times\frac{1}{2}=3$, $6\times3=18\times3=21$.

Multiply $4\frac{1}{3}$ by 8. $8\times\frac{1}{3}=2\frac{2}{3}$, $8\times4=32+2\frac{2}{3}=34\frac{2}{3}$.

Multiply $7\frac{1}{2}$ by 9. $9\times\frac{1}{2}=4\frac{1}{2}$, $9\times7=63+4\frac{1}{2}=67\frac{1}{2}$.

Multiply $8\frac{1}{2}$ by 12. $12\times\frac{1}{2}=6$, $12\times8=96+6=102$.

Multiply $7\frac{1}{4}$ by $7\frac{1}{4}$. $7\frac{1}{4}\times7\frac{1}{4}=52.\frac{9}{16}$

Multiply $7\frac{1}{2}$ by $7\frac{1}{2}=56\frac{1}{4}$.

Multiply $8\frac{2}{3}$ by $8\frac{1}{3}=72\frac{2}{9}$.

Multiply $9\frac{2}{7}$ by $9\frac{5}{7}=90\frac{10}{49}$.

DIVISION OF FRACTIONS.

RULE.—*Reduce mixed numbers to improper fractions, and whole numbers to the form of fractions; multiply the dividend by the divisor inverted.*

Invert the divisor to find how many times it is contained in one.

$\frac{8}{3}\times=\frac{18}{6}=2\frac{8}{6}$. Simply multiplying numerator and denominator by 2.

Divide $5\frac{1}{2}$ by $2\frac{1}{3}$. Multiply both numerator and denominator by 6, the least common multiple of 2 and 3.

Divide 25 by $\frac{1}{2}=50$.

Divide 21 by $3\frac{1}{3}=\frac{63}{10}=6\frac{3}{10}$.

To divide any number by $3\frac{1}{3}$, remove the point one place to the left and multiply by 3.

Divide 20 by $3\frac{1}{3}$. Remove the point one place we have 2, $2\times3=6$ Ans.

Divide 27 by $3\frac{1}{3}=8\frac{1}{10}$.

To divide any number by $2\frac{1}{2}$, remove the point one place to the left and multiply by 4.

Divide $20\frac{5}{10}$ by $2\frac{1}{2}$. Remove the point one place to the left and multiply by 4.

Removing the point one place to the left makes $2\frac{5}{100}$, $2\frac{5}{100}\times4=8\frac{1}{5}$ Ans.

To divide any number by $1\frac{1}{9}$, remove the point one place to the left and multiply by 9.

Divide 11 by $1\frac{1}{9}=9\frac{9}{10}$.

Divide any number by 5. Remove the point

one place and multiply by 2. Removing the point one place to the left divides the number by 10. In dividing by 10 we divide by a number twice too large; therefore we multiply by 2 for the correct result.

To divide any number by 12½, remove the point two places to the left and multiply by 8.

Divide 125 by 12½.

Divide 47·5 by 12½.

Divide 96 by 12½.

Divide 99 by 12½.

To divide any number by 25, remove the point two places to the left and multiply by 4.

To divide any number by 33⅓, remove the point two places to the left and multiply by 3.

To divide any number by 50, remove the point two places to the left and multiply by 2.

To divide by 66⅔, remove the point two places to the left divide by 2 and multiply by 3.

TO FIND THE VALUE OF CURRENCY WHEN GOLD IS AT A STATED PRICE.

When gold is $111\frac{1}{9}$, what is the value of $1.00 currency? We take the 100, the number of cents in a dollar, as the numerator, and the value of the gold as the denominator. Simplify the fraction by multiplying the numerator and de-

nominator by 9 and we have $\frac{9}{10}$ of a dollar or 90 cents; the value of the currency.

When gold is $109\frac{1}{8}$, what is the value of $1.00 currency?

$$\frac{100}{109} = \frac{900}{982} = \frac{450}{491} = \$.91\frac{319}{491}$$

When currency is worth 75 cents, what is the value of gold?

$$\frac{100}{75} = \frac{4}{3}, \ \frac{4}{3} \text{ of 100 cents equals } \$1.33\tfrac{1}{3}.$$

When gold is worth $105\frac{1}{2}$, what is the value of $1.00 currency?

$$\frac{100}{105\frac{1}{2}} = \frac{200}{211} = \$.94\frac{166}{211}$$

RULE.—*We take* 100, *the number of cents in a dollar, for the nuuerator, and the value of gold or currency, as the case may be, for the denominator. Simplify the fraction by annexing ciphers to the numerator and dividing by the denominator.*

INTEREST TABLE AND FORM FOR MAKING TABLES.

The following Table gives the Interest on any amount at 7 per cent., by simply removing the point to right or left, as the case may require:

Number of Days.	$100	$90	$80	$70
1	.0192	.01726	.01534	.01342
2	.0384	.03452	.03058	.02685
3	.0575	.05178	.04603	.04027
4	.0767	.06904	.06137	.05370
5	.0959	.08630	.07671	.06712
6	.1151	.10356	.09205	.08055
7	.1342	.12082	.10740	.09897
8	.1532	.13808	.12274	.10740
9	.1726	.15534	.13808	.12089
90	1.7260	1.5342	1.38082	1.20822
93	1.7836	1.60521	1.42685	1.24849
100	1.9178	1.82603	1.53425	1.24247

$60	$50	$40	$30	$20
.01151	.00950	.00767	.00575	.00384
.02301	.01918	.01534	.01151	.00767
.03452	.02877	.02301	.01726	.01151
.04603	.02836	.03068	.02301	.01536
.05753	.04795	.03836	.02877	.01918
.06904	.05753	.04603	.03452	.02313
.08055	.06712	.05370	.04027	.02685
.09205	.07671	.06137	.04603	.03068
1.0356	.08630	.06904	.05178	.03452
1.03562	.86301	.69041	.51781	.34521
1.07014	.89178	.71342	.53508	.35671
1.15065	.95890	.76712	.57534	.48356

To Find the Difference of Time between Two Dates by the following Table:

Rule.—*Opposite the day of the month is written the number of days of the year which have expired. Subtract this number from the whole number of days that have expired at the last date.*

Thus: What is the time from the first day of March to the 27th day of September? The 1st day of March we find by the table that 60 days of the year are gone. The 27th day of September we find that 270 days are gone. Hence 270 days minus 60 days equals 210 days, the time between the two dates.

Rule for Examining the Date of Notes, Deeds, &c., and Telling the Day of the Month.

Rule.—Take 7 for the unity term of a number, and the day of the month, plus the excess of sevens in the ratios of the century, year and month, for the unit term, rejecting the sevens in the statement, the excess is the day of the week, 1 of excess is Sunday, 2 Monday, 3 Tuesday, &c., 0 excess indicates Saturday.

Example.—What day of the week will the 4th of July 1878 occur?

Statement.— $\frac{4+1}{7}$ = 5th, or Thursday, ans.

All examples are performed the same. See p. 197.

3 January		6 February		6 March		2 April		4 May		0 June	
1	1	1	32	1	60	1	91	1	121	1	152
2	2	2	33	2	61	2	92	2	122	2	153
3	3	3	34	3	62	3	93	3	123	3	154
4	4	4	35	4	63	4	94	4	124	4	155
5	5	5	36	5	64	5	95	5	125	5	156
6	6	6	37	6	65	6	96	6	126	6	157
7	7	7	38	7	66	7	97	7	127	7	158
8	8	8	39	8	67	8	98	8	128	8	159
9	9	9	40	9	68	9	09	9	129	9	160
10	10	10	41	10	69	10	100	10	130	10	161
11	11	11	42	11	70	11	101	11	131	11	162
12	12	12	43	12	71	12	102	12	132	12	163
13	13	13	44	13	72	13	103	13	132	13	164
14	14	14	45	14	73	14	104	14	134	14	165
15	15	15	46	15	74	15	105	15	135	15	166
16	16	16	47	16	75	16	106	16	136	16	167
17	17	17	48	17	76	17	107	17	137	17	168
18	18	18	49	18	77	18	108	18	138	18	169
19	19	19	50	19	78	19	109	19	139	19	170
20	20	20	51	20	79	20	110	20	140	20	171
21	21	21	52	21	80	21	111	21	141	21	172
22	22	22	53	22	81	22	112	22	142	22	173
23	23	23	54	23	82	23	113	22	143	23	174
24	24	24	55	24	83	24	114	24	144	24	175
25	25	25	56	25	84	25	115	25	145	25	176
26	26	26	57	26	85	26	116	26	146	26	177
27	27	27	58	27	86	27	117	27	147	27	178
28	28	28	59	28	87	28	118	28	148	28	179
29	29			29	88	29	119	29	149	29	180
30	30			30	89	30	120	30	150	30	181
31	31			31	90			31	151		

2		5		1		3		6		1	
July		August		September		October		November		December	
1	182	1	213	1	244	1	274	1	305	1	335
2	183	2	214	2	245	2	275	2	306	2	336
3	184	3	215	3	246	3	276	3	307	3	337
4	185	4	216	4	247	4	277	4	308	4	338
5	186	5	217	5	248	5	278	5	309	5	339
6	187	6	218	6	249	6	279	6	310	6	340
7	188	7	219	7	250	7	280	7	311	7	341
8	189	8	220	8	251	8	281	8	312	8	342
9	190	9	221	9	252	9	282	9	313	9	343
10	191	10	222	10	253	10	283	10	314	10	344
11	192	11	223	11	254	11	284	11	315	11	345
12	193	12	224	12	255	12	285	12	316	12	346
13	194	13	225	13	256	13	286	13	317	13	347
14	195	14	226	14	257	14	287	14	318	14	348
15	196	15	227	15	258	15	288	15	319	15	349
16	197	16	228	16	259	16	289	16	320	16	350
17	198	17	229	17	260	17	290	17	321	17	351
18	199	18	230	18	261	18	291	18	322	18	352
19	200	19	231	19	262	19	292	19	323	19	353
20	201	20	232	20	263	20	293	20	324	20	354
21	202	21	233	21	264	21	294	21	325	21	355
22	203	22	234	22	265	22	295	22	326	22	356
23	204	23	235	23	266	23	296	23	327	23	357
24	205	24	236	24	267	24	297	24	328	24	358
25	206	25	237	25	268	25	298	25	329	25	359
26	207	26	238	26	269	26	299	26	330	26	360
27	208	27	239	27	270	27	300	27	331	27	361
28	209	28	240	28	271	28	301	28	332	28	362
29	210	29	241	29	272	29	302	29	333	29	363
30	21	30	242	30	273	30	303	30	334	30	364
31	21	31	243			31	304			31	365

POWERS AND ROOTS.

The product of a number taken any number of times as a factor, is called a power of the number.

A root of a number is such a number as taken some number of times as factor will produce a given number.

If the root is taken twice as a factor to produce the number, it is the square root. If three times, the cube root. If four times, the fourth root, etc.

ILLUSTRATION.—5 is the square root of 25. The cube root of 125. The fourth root of 625, because $(5)^2=25$, $(5)^3=125$, $(5)^4=625$.

$(1)^2=1$	$(1)^3=1$
$(2)^2=4$	$(2)^3=8$
$(3)^2=9$	$(3)^3=27$
$(4)^2=16$	$(4)^3=64$
$(5)^2=25$	$(5)^3=125$
$(6)^2=36$	$(6)^3=216$
$(7)^2=49$	$(7)^3=343$
$(8)^2=64$	$(8)^3=512$
$(9)^2=81$	$(9)^3=729$
$(10)^2=100$	$(10)^3=1000$

We observe that the square of any one of the digits is less than 100. And the cube of any one of the digits is less than 1000. Hence the square root of two figures cannot give more than one figure.

Hence if we begin at the right of any number and separate it into periods of two figures each, the number of periods would be the same as the number of figures in its square root.

In order to understand the method of extracting square root, it is necessary to consider how the square of a number consisting of two parts is formed from those parts.

To do this let a represent any number whatever, b represent any other number, then will $a+b$ represent the sum and $(a+b)^2$ the square of the sum of any two numbers, but since the square of any two terms is the square of the first, plus two times the first into the second, plus the square of the second: we have $(a+b)^2=a^2+2ab+b^2$.

ILLUSTRATION. — 23 here $a=20$ and $b=3$. Hence $(a+b)^2$ will equal $(20+3)^2$. In applying the above formula, commence at the units instead of the tens to find the square of the number. Thus 3^2 is 9, two times 3 into 2 is 12. Write down the 2 and carry the 1 to the square of the first term 2, and we have 529, the square of 23 and 23 is the square root of 529.

The square of any number of terms is the square of the first, plus two times the first into the second, plus the square of the second, plus two times the sum of the first two into the third, plus the square of the third, plus two the sum of the first three into the fourth, plus the square of

the fourth, etc. Note—In applying the above formula commence at the units to square numbers.

Method of Extracting Square Root.

625. This number contains two periods; hence there are two figures in the root. The greater square below 6, the first or left hand period is 4, the root of which is 2; and since there are two figures in the root, 2 will stand in the tens place and equal 20. Hence, we subtract the square of 20, which is 400, from 625, and we have 225 remaining. We have found a square 20 feet on a side. Now, in order to preserve the square, we make the addition on two adjacent sides. Hence, we double 20, the length of one side, and get 40, the trial divisor; dividing 225 by 40, we get the width of the addition, 5 feet; adding 5 feet to 40 feet, the width of the little square in the corner, we get 45, the true divisor. Multiplying 45 by 5, we get 225, the surface of the addition. Hence, 25 is the length of one side of a square that contains 625 square feet.

	100	3 2 1 15625(100+20+5.
1st trial divisor	200	10000
1st true "	220	5625
2d trial "	240	4400
		1225
2d true "	245	1225

DECIMAL METHOD OF EXTRACTING SQUARE ROOT.

RULE.—Remove the decimal point two places to the left in any number, extract the square root of the quotient, and we have one tenth of the root of the number.

Remove the decimal point *four* places to the left in any number, extract the square root of the quotient and we have one hundredth part of the root of the number.

Remove the decimal point *six* places to the left in any number, extract the square root of the quotient, and we have the thousandth part of the root of the number.

What is the square root of 9604?
What is the square root of 2401?
What is the square root of 225?
What is the square root of 64?

CUBE ROOT.

Relation of Cube to Root.

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 739$
$10^3 = 1000$

By observation we see that the entire part of the cube root of any number below 1000 will be less than 10, and will, therefore, contain but one figure. The entire part of the cube root of a number containing four, five or six figures, will contain two figures, and so on with the larger numbers.

A cube of any number of terms, is the cube of the first term, plus three times the square of the first into the second, plus three times the first into the square of the second, plus the cube of the second, plus three times the square of the sum of the first two into the third, plus three times the sum of the first two into the square of the third, plus the cube of the third, etc.

HENDERSON'S DECIMAL METHOD

OF

EXTRACTING THE CUBE ROOT,

ABBREVIATING THE NEWTONIAN METHOD.

$$(a+b+c+d)^3 = a^3+3a^2b+3ab^2+b^3+3(a+b)^2c+3(a+b)c^2+c^3+3\,(a+b+c)^2d+3(a+b+c)d^2+d^3.$$

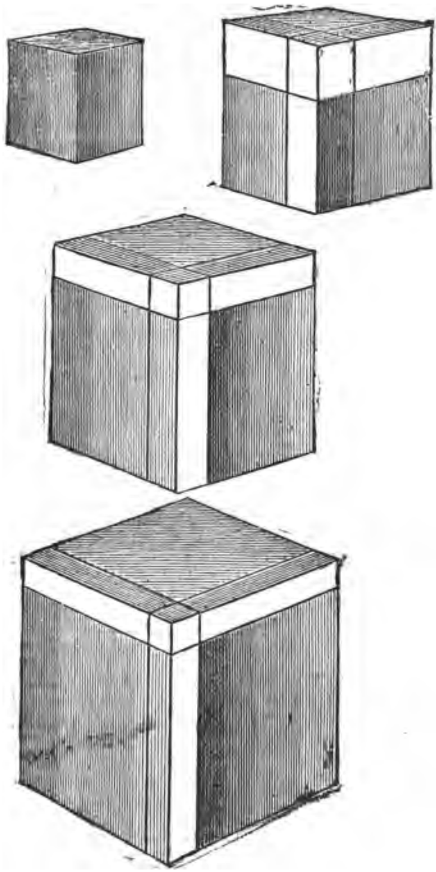
1000+100+20+ 5			1.423828125
			1000000000
a^2=1000000		1st Dividend, =	423828125
$3a^2$=3000000=	1st Trial Divisor.		331000000
3ab= 300000		2d Dividend, =	92828125
b^2= 10000			73928000
3310000=	1st True Divisor.	3d Dividend, =	18900125
300000			18900125
20000			0
3630000=	2d Trial Divisor.		
3(a+b) c=66000			
b^2= 400			
3695400=	2d True Divisor.		
66000			
800			
3763200=	3d Trial Divisor.		
3(a+b+c)d=16800			
d^2= 25			
3780205=	3d True Divisor.		

Applying this example to the engraving illustrating the decimal method of extracting roots, 1000 is the linear edge of the first cube, 1000000 is the surface of one side, 1000000000 the solid contents.

In the second cube the dark parts represent the surface contained in the divisor, while the blanks represent what is necessary to add to find the next trial divisor. In the third and fourth cubes the same remarks apply, thus saving by this method the usual labor of squaring the root and multiplying by 3 for each trial divisor.

RULE.—Remove the decimal point three places to the left, extract the cube root of the quotient, and we have one-tenth of the root. Remove the decimal point six places to the left, extract the cube root of the quotient, and we have one-hundredth part of the root of the given number. Remove the point nine places to the left, extract the cube root of the quotient, and we have one-thousandth part of the root.

Removing the point 3 times 3, or 9 places to the left, we find that one of the three equal factors of one billion is one thousand. We place it to the left for the first figure of the root, 1000 feet, is the linear edge of a cube that contains one billion cubic feet. The first trial divisor is the surface of the three adjacent sides of this cube, or three millions square feet. We find the divisor is contained in the first dividend 100 times. We find three rectangles



Rule of Cube Root.

The sum of the three sides of the complete cube invariably represents the trial divisor. To find each *trial divisor*, or the three sides of the complete cube, add what is shown as vacant in the engraving to the last true divisor. Each TRUE DIVISOR is found by adding to the trial divisor three times the surface of one side of each parallelopiped, and one side of the small cube.

N. B.—Each figure in the root of any number, after the first, as it occurs, equals the length of the small cube, or thickness of the addition; the root without this figure is the length of each parallelopiped.

Second Rule of Cube Root

Divide any number by any cube, extract the cube root of the quotient, and multiply this root by the linear edge of the cube used as a divisor, and we have the cube root of the number.

Rule of Square Root.

Divide any number by a square, extract the square root of the quotient and multiply the root found by the linear edge of the square used as a divisor, and we have the square root of the number.

The above rules are universal in their application to all examples.

and a small cube necessary to complete the first addition. The length of one is 1000 feet, three 3000 feet, the width being 100 feet, the surface of the three sides is 300000 square feet, and the small cube the square of 100 feet, or 10000 square feet. Hence, the sum of the surfaces of one side of each of the pieces necessary to complete the addition makes the true divisor, or 3310000; multiplying this number by 100, the thickness of the addition, we find the number of cubic feet in the first addition, which subtracted from the first dividend leaves a second dividend of 90828125 cubic feet to be added. In order to preserve the cubical form we make the additions on three adjacent sides, which forms the second trial divisor. We observe that the surface of these three sides is found by bringing down the 300000 and doubling the 10000, the surface of one side of the small cube, and adding to the first true divisor we get three sides of the cube, or the second trial divisor. We find it contained in the second dividend 20 times. Adding to this trial divisor the surface of one side of each of the three rectangles and one side of the small cube, we get the second true divisor. Multiplying this true divisor by 20, the thickness, we get the solid contents of the second addition, which deducted fron the second dividend, leaves a third dividend of 18900125 to be added. Bringing down 66000, the surface of one side of each of the three rectangles in the last addition and 800, two sides of the small cube, we have the third trial divisor, or

three sides of the cube. We find this divisor is contained in the third dividend 5 times. Adding the surface of one side of each of the three rectangles, or 16800 square feet and 25 feet, the surface of one side of the small cube, we have 3780025 square feet, the third and last true divisor, which multiplied by five feet, the thickness, gives 18900125 cubic feet, or the solid contents of the last addition.

2d Solution.—1423828125÷1953125 = 729, and the cube root of 729 is 9 and 9 times 125, the linear edge of the cube used as the divisor equals 1125, the cube root of the number.

3d Solution.—1423828125÷15625=91125, and the cube root of 91125 is 45, which multiplied by 25, the linear edge of the cube used as the divisor equals 1125, the cube root of the number.

4th Solution.—1423828125 divided by the cube of 45 gives 15625, the cube root of which is 25, and 25×45, the linear edge of the cube used as a divisor, gives 1125, the cube root of the number.

What is the cube root of 59319?

Presume the root to be divided into 13 equal parts.

What is the cube root of 117649?

Presume the root to be divided in 7 equal parts.

What is the cube root of 97336?

Presume the root to be divided in 23 equal parts.

What is the cube root of 95112?

Let the root be divided into 29 equal parts.

The number divided by the cube of 29 equals 8, and the cube root of 8 is 2. Hence 29 times 2 is the cube root of the number or 58.

What is the cube root of 91125?

Let the root be divided into 9 equal parts, the number divided by the cube of 9 equals 125, the cube root of 125 is 5. Hence 9 times 5 or 45 is the cube root of the number.

What is the cube root of 216×343?

The cube root of 216 is 6. The root of 343 is 7.

The cube root of the product is 6 times 7 or 42.

What is the cube root of 64×125?

The cube root of 64 is 4. The cube root of 125 is 5, $5\times4=20$. The cube root of the product.

What is the cube root of $125\times125=5\times5$?

What is the cube root of $125\times15625=5\times25$

What is the cube root of 512×729?

The cube root of 512 is 8. The cube root of 729 is 9.

The cube root of the product is 8×9 or 72.

What is the cube root of 216×729?

The cube root of 216 is 6. The cube root of 729 is 9.

The cube root of the product is 6×9 or 54.

SQUARE AND CUBE ROOT OF FRACTIONS.

To square a fraction, we square its numerator for the numerator, and its denominator for the denominator. Hence, to find the square root of a fraction, we must extract the square root of its numerator, for the numerator of the answer, and the square root of its denominator for the denominator of the answer.

ILLUSTRATIONS.—Find the square root of $\frac{4}{9}$. The square root of 4, the numerator is 2. The square root of 9, the denominator, is 3. Hence the answer, $\frac{2}{3}$.

What is the square root of $\frac{25}{100} = \frac{5}{10}$.

What is the square root of .0081 = .09.

When both terms of the fraction are not perfect squares, only an approximate value of the root can be obtained.

In order that the denominator of a decimal fraction may be a perfect square, its numerator must contain an even number of decimal places. Hence, to extract the square root of a decimal fraction, make its number of decimal places even, by annexing a zero, if necessary; extract the root, as in whole numbers, observing that there will be one decimal place in the root for every two in the given fraction, the root may be found to any number of decimal places by annexing two zeros for every additional figure.

To extract the cube root of a fraction, we extract the cube root of the numerator for the numerator of the answer, and the cube root of the denominator for the denominator of the answer. If its numerator and denominator are not perfect cubes, the approximate value of the cube root can only be obtained. If the denominator is not a perfect cube, both terms should be multiplied by the square of the denominator. Hence, to extract the square root of a decimal fraction, annex zeros, if necessary, to make its number of decimal places some multiple of three; extract its root, as in whole numbers, observing that there will be one decimal place for every three in the given fraction.

TO FIND THE SURFACE OF PLANE FIGURES.

A triangle is a figure having three sides and three angles.

The altitude of a triangle is the perpendicular distance from the side assumed as its base to to the vertex of the opposite angle.

RULE.—*To find the surface of any triangle, multiply the base by half the altitude.*

A right-angle triangle is a triangle having a right angle.

Lines are parallel when they lie in the same direction. A parallelogram is a four-sided figure having its opposite sides parallel.

A trepizoid is a four-sided figure, having two of its sides parallel.

A polygon is a figure bounded on all sides by straight lines.

Similar figures are those which have the same shape.

The corresponding sides are proportional.

The base of a figure is the side on which it is supposed to stand.

The altitude of a rectangle, a parallelogram or a trepizoid, is the perpendicular distance between its parallel basis.

The area of a rectangle is the length multiplied by the width.

Method of Measuring Land.

Find the number of rods by multiplying the length by the width. Remove the point two places to the left, divide by eight and multiply the quotient by five; or remove the point two places, take $\frac{5}{8}$ of the result, and we have the number of acres. Thus: 3280 rods, the point removed two places leaves $32.80 \div 8 = 4.1$. $4.1 \times 5 = 20.5$ acres.

What is the number of acres in 2440 rods? Remove the point two places we have 24.40; $\frac{5}{8} \times 24.40$ is $15\frac{1}{4}$, the number of acres. This method is of universal application, and may be stated in the following words: *Remove the decimal point two places to the left, and $\frac{5}{8}$ of the quotient are the number of acres.*

We remove the point two places to reduce the number to units of a hundred, and since there are $\frac{8}{5}$ of a hundred rods in one acre, five times $\frac{1}{8}$ of the number of hundred rods must equal the number of acres; or simply the point removed two places and the quotient divided by $\frac{8}{5}$ equals the number of acres.

What are the number of acres in a field 160 rods wide and 480 rods long? Remove the point two places on 160, and take $\frac{5}{8}$ of the quotient, we find one acre multiplied by 480, the length, we get 480 acres, Ans.

What is the number of acres in a field 2200 rods long aud 640 wide?

What is the number of acres in a field of triangular shape? The base of the triangle is 800 rods and the altitude 300; since the area is the base multiplied by half the altitude. Half the altitude is 150; remove the point two places on 800, and we have 8, and $\frac{5}{8}\times8=5$, and $5\times150=750$, the number of acres in the field.

The area of a circle also equals the square of its radius multiplied by 3.1416, the ratio of the circumference to the diameter. If the radius is two feet the area of the circle is $3.1416\times2^2=12.5664$.

Find the area of a circle 12 fect in diameter.

Find the area of a circle of 8 feet radius; of a circle of 100 feet radius.

The suface of a sphere equals the square of its diameter multiplied by 3.1416.

ILLUSTRATION.—The surface of a sphere 5 feet in diameter$=3.1416\times25$.

The surfaces of spheres are to each other as the squares of their diameters.

The solidity of a sphere equals the product of the surface multiplied by $\frac{1}{6}$ of the diameter, or it equals $\frac{1}{6}$ of the cube of the diameter multiplied by 3.1416. The solidities of spheres are to each other as the cubes of their diameters.

The solidities of similar solids are to each other as the cubes of their like dimensions.

The solidity of a cylinder equals the product of the area of its base by its altitude.

The convex surface of a cylinder equals the product of the circumference of its base by its altitude.

What is the solidity of a cylinder 8 feet high with a base 4 feet in diameter? A cylinder 12 feet high, with a base 1 foot diameter?

What is the diameter of a sphere containing 100 cubic feet?

HENDERSON'S

NEW DECIMAL METHOD OF MEASURING GRAIN AND LIQUID

RULE.—Having found the number of cubic feet in a box or bin, remove the decimal point one place to the left in the sum found and multiply by 8 in all examples; because, a cubic foot is eight-tenths of a bushel nearly. Add $4\frac{1}{2}$ bushel for every 1000 bushels so found for correct answer.

To find the number of gallons, multiply the number of bushels by 8.

EXAMPLE—.—Suppose a box or bin to be 75 feet long, 56 feet wide, and 27 feet deep, thus, $75 \times 56 \times 27 = 113400$ cubic feet: To find the number of bushels, remove the decimal point one place to the left, and multiply by 8.

Some of the Miscellaneous Weights to the Bushel.

60	lbs make	1	bushel of	Wheat.
56	"	1	"	Corn.
33	"	1	"	Oats.
48	"	1	"	Barley.
56	"	1	"	Rye.
60	"	1	"	Beans.
52	"	1	"	Buckwheat.
70	"	1	"	Corn in ear.
50	"	1	"	Corn meal.
60	"	1	"	Potatoes.
50	"	1	"	Salt.
33	"	1	"	Peaches, dried.
25	"	1	"	Apples, dried.
62	"	1	"	Clover seed.
45	"	1	"	Timothy.
56	"	1	"	Flax.

SHORT METHODS IN DIVISION AND MULTIPLICATION.

Remove the point one place to the right to multiply by 10; two places to multiply by 100; three places 1000, etc.

To divide, remove it to the left.

To multiply by 25, divide by 4 and call the quotient hundreds.

Thus: 25×480=12000. 480÷4=120 call it hundreds, makes 12000. Divide by 4, because 25 is one quarter of a hundred.

To multiply by 2½ divide by 4 and call it tens; call it tens, because 2½ is the quarter of ten.

To multiply by 125, divide by 8 and call it thousands. Call it thousands, because 125 is $\frac{1}{8}$ of a thousand.

To multiply by $12\frac{1}{2}$ divide by 8; call it hundreds.

To multiply by $1\frac{1}{4}$ divide by 8; call it tens.

To multiply by $62\frac{1}{2}$ divide by 16 and call it thousands.

To multiply by $6\frac{1}{4}$ divide by 16 and call it hundreds.

To multiply by $31\frac{1}{4}$ divide by 32 and call it thousands.

To multiply by $333\frac{1}{3}$, divide by 3 and call it thousands.

To multiply by $33\frac{1}{3}$, divide by 3 and call it hundreds.

To multiply by $3\frac{1}{3}$, divide by 3 and call it tens.

To multiply by 50, divide by 2 and call it hundreds.

To multiply by $66\frac{2}{3}$, divide by 15 and call it thousands.

To multiply by $6\frac{2}{3}$, divide by 15 and call it hundreds.

To multiply by $833\frac{1}{3}$, divide by 12 and call it ten thousands, by annexing four ciphers.

To multiply by $83\frac{1}{3}$, divide by 12 and call it thousands.

To multiply by $8\frac{1}{3}$, divide by 12 and call it hundreds. Divide by 12 and call it hundreds.

because $8\frac{1}{3}$ is $\frac{1}{12}$ of a hundred.. The reason is similar in each case.

The primitive meaning of reason is hook something to hold on by. Please get the reason in each case.

To multiply by $166\frac{2}{3}$, divide by 6 and call it thousands; because $166\frac{2}{3}$ is $\frac{1}{6}$ of 1000.

To multiply by $16\frac{2}{3}$, divide by 6 and call it hundreds.

To multiply by $1\frac{2}{3}$, divide by 6 and call it tens.

To multiply by $37\frac{1}{2}$, take $\frac{3}{8}$ of the number and call it hundreds; $87\frac{1}{2}$, $\frac{7}{8}$ of the number, and call it hundreds, etc.

We simply reverse these methods to divide.

To divide by 10, 100, 1000, etc., we remove the point one, two, and three places to the left.

To divide by 25, remove the decimal point two places to the left and multiply by 4.

Removing the point two places divides by one hundred; hence the quotient is 4 times to small; hence we remove the point two places and multiply by 4.

To divide by $2\frac{1}{2}$, remove the point one place to the left and multiply by 4.

To divide by 125, remove the point three places to the left and multiply by 8.

To divide by $12\frac{1}{2}$, remove the point two places to the left and multiply by 8.

To divide by $1\frac{1}{4}$, remove the point one place to the left and multiply by 8. There are about

$1\frac{1}{4}$ cubic feet in one bushel. Hence divide the number of cubic feet by $1\frac{1}{4}$ gives the number of bushels nearly.

To divide by 625, remove the point four places to the left and multiply by 16.

To divide by $62\frac{1}{2}$, remove the point three places to the left and multiply by 16.

To divide by $6\frac{1}{4}$, remove the point two places to the left and multiply by 16.

To divide by 3125, remove the point five places to the left and multiply by 32.

To divide by $3\frac{1}{8}$, remove the point two places to the left and multiply by 32.

To divide $333\frac{1}{3}$, remove the point three places to the left and multiply by three.

To divide by $666\frac{2}{3}$, remove the point four places to the left and multiply by 15.

To divide by $66\frac{2}{3}$, remove the point three places to the left and multiply by 15.

To divide by $833\frac{1}{3}$, remove the point four places to the left and multiply by 12.

To divide by $83\frac{1}{3}$, remove the point three places to the left and multiply by 12.

To divide by $8\frac{1}{3}$, remove the point two places to the left and multiply by 12.

To divide by $166\frac{2}{3}$, remove the point three places to the left and multiply by 6. Removing the point three places divides by 1000; hence the quotient is 6 times too small. $166\frac{2}{3}$ is $\frac{1}{6}$ of 1000.

MENTAL EXERCISE.

PROBLEM 1.—Take 1, multiply by 49, extract the square root, multiply by 4, subtract 1, and extract the cube root; what is the result?

PROBLEM 2.—Take 9, divide by 2, multiply by 6, extract the cube root, multiply by 27, and extract the fourth root; what is the result?

PROBLEM 3.—Take 48, divide by 2, multiply by 4, add 4, extract the square root, multiply by 5, subtract 1, divide by seven, and what is the result?

PROBLEM 4.—Take $8\frac{2}{3}$, multiply by $8\frac{1}{3}$, subtract $\frac{2}{9}$, divide by 8, extract the square root, multiply by 40 and divide by 10; what is the result?

PROBLEM 5.—Take $1\frac{1}{2}$, multiply by $1\frac{1}{2}$, $2\frac{1}{2}$ by $2\frac{1}{2}$, $3\frac{1}{2}$ by $3\frac{1}{2}$, run it up to $12\frac{1}{2}$, in concert.

PROBLEM 6.—Take $1\frac{1}{3}$, multiply by $1\frac{2}{3}$, $2\frac{1}{3}$ by $2\frac{2}{3}$, etc., up to 12.

PROBLEM 7.—Take $1\frac{2}{5}$, multiply by $1\frac{3}{5}$, $2\frac{2}{5}$ by $2\frac{3}{5}$, etc., up to 15.

PROBLEM 8.—Take $1\frac{3}{7}$, multiply by $1\frac{4}{7}$, $2\frac{3}{7}$ by $2\frac{4}{7}$, etc., up to 20.

PROBLEM 9.—Take $1\frac{3}{8}$, multiply by $1\frac{5}{8}$, $2\frac{3}{8}$ by $2\frac{5}{8}$, etc., up to 17.

PROBLEM 10.—Take $1\frac{4}{9}$, multiply by $1\frac{5}{9}$, $2\frac{4}{9}$ by $2\frac{5}{9}$, etc.

PROBLEM 11.—Take $1\frac{5}{12}$, multiply by $1\frac{7}{12}$, $2\frac{5}{12}$ by $2\frac{7}{12}$, etc.

PROBLEM 12.—Take $1\frac{7}{11}$, multiply by $1\frac{4}{11}$, $2\frac{7}{11}$ by $2\frac{4}{11}$, etc.

PROBLEM 13.—Take $12\frac{1}{2}$, multiply by $12\frac{1}{2}$, $11\frac{1}{2}$ by $11\frac{1}{2}$, etc., down to 1.

PROBLEM 14.—Take $11\frac{1}{4}$, multiply by $11\frac{3}{4}$, $10\frac{1}{4}$ by $10\frac{3}{4}$, etc., down to 1.

PROBLEM 15.—Take $12\frac{7}{8}$, multiply by $12\frac{1}{8}$, $11\frac{7}{8}$ by $11\frac{1}{8}$, etc., down to 1.

PROBLEM 16.—Take $13\frac{3}{5}$, multiply by $13\frac{2}{5}$, $12\frac{3}{5}$ by $12\frac{2}{5}$, etc., down to 1.

PROBLEM 17.—Take $12\frac{9}{10}$, multiply by $12\frac{1}{10}$, $11\frac{9}{10}$ by $11\frac{1}{10}$, etc., down to 1.

PROBLEM 18.—Take $10\frac{7}{13}$, multiply by $10\frac{6}{13}$, etc., down to 1.

PROBLEM 19.—Take $12\frac{7}{12}$, multiply by $12\frac{5}{12}$, etc., down to 1.

PROBLEM 20.—Take $8\frac{7}{15}$, multiply by $8\frac{8}{15}$, $7\frac{7}{15}$ by $7\frac{8}{15}$, etc., down to 1.

PROBLEM 21.—Take $10\frac{9}{16}$, multiply by $10\frac{7}{16}$, $9\frac{9}{16}$ by $9\frac{7}{16}$, etc., down to 1.

PROBLEM 22.—Take $12\frac{9}{14}$, multiply by $12\frac{5}{14}$, etc., down to 1.

PROBLEM 23.—Take $11\frac{9}{20}$, multiply by $11\frac{11}{20}$, etc., down to 1.

PROBLEM 24.—Take $12\frac{1}{9}$, multiply by $12\frac{8}{9}$, etc., down to 1.

PROBLEM 25.—Take $8\frac{7}{19}$, multiply by $8\frac{12}{19}$, etc., down to 1.

PROBLEM 26.—Take $13\frac{4}{7}$, multiply by $13\frac{3}{7}$; etc., down to 1.

The mean of two numbers is half their sum, or the number equally distant from the two numbers.

The product of two numbers is the square of their mean diminished by the square of half of their difference.

PROBLEM 27.—19 times 21, 18 times 22, etc., down to 15. Thus: The mean is 20, the square of 20 is 400, 400—the square of 1 is 399; the product, 18 times 22 is the square of 20, 400—the square of 2, 4, 396. 17 times 23 is 391, 16 times 24, 384; 15 times 25, 375.

PROBLEM 28.—Take 29 by 31, 28 by 32, etc., down to 20 and up to 40.

PROBLEM 29.—Take 39 by 41, 38 by 42, etc., down to 30 and up to 50.

PROBLEM 30.—Take 49 by 51, 48 by 52, etc., down to 40 and up to 60.

PROBLEM 31.—Take 59 by 61, 58 by 62, etc., down to 50 and up to 70.

PROBLEM 32.—Take 69 by 71, 68 by 72, etc., down to 60 and up to 80.

PROBLEM 33.—Take 79 by 81, 78 by 82, etc., down to 70 and up to 90.

PROBLEM 34.—Take 89 by 91, 88 by 92, etc., down to 90 and up to 100.

The complement of a number is the difference of that number and some particular number above it. The supplement of a number is the difference of that number and some particular number below it.

Thus, the complement of 99 is the difference of 99 and 100, which is 1.

The supplement of 101 is the difference of 101 and 100, which is 1.

PROBLEM 35.—Commence at 99 and square numbers down to 90. Thus: 99 times 99 is 9801, 98 times 98 is 9604, 97 times 97 is 9409, 96 times 96 is 9216, etc. Simply diminish the number by its complement, call it hundreds and add the square of the complement.

When we use the supplement, we add it to the number, give it its proper name and add the square of the supplement.

Thus: 101 times 101, the supplement 1 added to 101 makes 102, call it hundreds, is 10200, plus the square of the supplement is 10201.

PROBLEM 36.—Commence at 101, square all the numbers up to 110 and down to 90.

PROBLEM 37.—Commence at 51, square all the numbers up to 60 and down to 40.

PROBLEM 38.—Commence at 21, square all the numbers up to 25 and down to 15.

PROBLEM 39.—Commence at 11, square all the numbers up to 15 and down to 5.

PROBLEM 40.—Commence at 999, square all the numbers down to 990 and up to 1010, etc., etc., etc., etc.

MISCELLANEOUS PROBLEMS.

PROBLEM 1.—How many bushels in a bin 10 feet long, 4 feet wide and 4 feet deep?

SOLUTION.—Since there are $\frac{10}{8}$ of a cubic foot in one bushel, the bin will contain 8 times $\frac{1}{10}$ of the number of cubic feet, in bushels. $\frac{1}{10}$ of 10 is 1, 8 times 1 are 8, 4 times 8, 32, and 4 times 32, 128, Ans. Or find the number of cubic feet in the bin, remove the decimal point one place to the left, and multiply by 8 in all cases. Thus: the product of 4, 4 and 10 is 160; remove the point one place to the left and we have 16, 16 multiplied by 8 is 128, Ans.

PROBLEM 2.—How many bushels in a bin 32 feet long, 16 feet wide and $5\frac{1}{2}$ feet high?

PROBLEM 3.—How many bushels in a bin 24 feet long, 12 feet wide, $4\frac{1}{2}$ feet high?

PROBLEM 4.—A cubic foot of water weighs 62 lbs. 8 oz.—what is the pressure on 5 acres at the bottom of the sea, where the water is 1 mile deep?

PROBLEM 5.—What would be the weight of this planet if one cubic foot weighs $62\frac{1}{2}$ pounds?

PROBLEM 6.—If $21\frac{3}{4}$ bushels of oats are required to seed $9\frac{3}{8}$ acres, how many bushels will be required to seed a field of 100 acres?

PROBLEM 7.—If $33\frac{1}{3}$ pounds of tea cost $\$27\frac{1}{2}$, how much will 300 pounds cost?

PROBLEM 8.—A field $3\frac{1}{2}$ times as long as it is wide contains 30 acres—what are its dimensions?

PROBLEM 9.—If each one of 20 pupils breathe 30 cubic feet of air per hour, in how long a time will they breathe as much air as a room 20 by 30 and 8 feet high contains?

PROBLEM 10.—If gold is $1.12\frac{1}{2}$, what is currency worth?

SOLUTION.—The value of currency would be $\frac{100}{112\frac{1}{2}}$, simply multiplying the numerator and denominator by 2 and we have $\frac{200}{225}=\frac{8}{9}$; hence one dollar in currency is worth $\frac{8}{9}\times\frac{100}{1}$ cents, or $88\frac{8}{9}$ cents.

PROBLEM 11.—If currency is worth $88\frac{8}{9}$ cents on the dollar, what is gold worth? Simply invert the preceding operation.

PROBLEM 12.—If gold is $1.10\frac{1}{2}$, what is currency?

PROBLEM 13.—If currency is 95 cents on the dollar, what is gold?

PROBLEM 14.—If a wolf can eat a sheep in $\frac{7}{8}$ of an hour, and a bear in $\frac{3}{4}$ of an hour, how long will it take them together to eat what remains of a sheep after the wolf has been eating half an hour?

SOLUTION.—In one hour the wolf eats $\frac{8}{7}$ of a sheep, after eating half an hour $\frac{3}{7}$ of the sheep would remain, since in one hour they eat $\frac{8}{7}+\frac{4}{3}$ or $\frac{52}{21}$; to eat $\frac{3}{7}$ or $\frac{9}{21}$ of a sheep it would take them as long as $\frac{52}{21}$ is contained in $\frac{9}{21}$, which is $\frac{9}{52}$ of an hour, Ans.

PROBLEM 15.—John cuts a cord of wood in $\frac{3}{4}$ of a day, James in $\frac{5}{6}$ of a day, how long will it take them to cut a cord when they work together?

PROBLEM 16.—A can do a piece of work in 8 days and A and B can do the same in 5 days; after A did $\frac{1}{3}$ of the work, B did the remainder —how long did it take him?

PROBLEM 17.—Divide the number 108 into two such parts, that $\frac{3}{7}$ of the first $+$ 8 shall equal the second.

PROBLEM 18.—A ship mast 63 feet in length, in a storm, was broken off; $\frac{2}{3}$ of what was broken off equaled $\frac{3}{4}$ of what remained; how much was broken off, and how much remained?

PROBLEM 19.—A farmer has 2290 sheep in two fields, $\frac{3}{4}$ of the number in the first field equals $\frac{2}{7}$ of the number in the second; how many are there in each field?

PROBLEM 20.—A market woman was requested to buy 99 fowls, consisting of two different kinds; $\frac{1}{4}$ of the number of the first kind was to equal $\frac{2}{3}$ of the second kind; how many of each kind must she buy?

PROBLEM 21.—A farmer, after selling $\frac{2}{3}$ of $1\frac{1}{5}$ times as much grain as he had, had 100 bushels remaining; how much had he at first?

PROBLEM 22.—Divide the number 170 into two parts, that shall be to each other as $\frac{2}{3}$ to $\frac{3}{4}$.

PROBLEM 23.—$\frac{2}{3}$ of A's number of sheep plus

$\frac{3}{4}$ of B's number equals 900; how many sheep has each, providing $\frac{3}{4}$ of B's number is $\frac{4}{5}$ of A's number?

PROBLEM 24.—A gold and silver watch were bought for $320; the silver watch cost $_7$ as much as the gold one; what was the cost of each?

PROBLEM 25—$\frac{1}{2}$ of A's money + $\frac{2}{3}$ of B's; equals 6600; and $\frac{2}{3}$ of B's is 4 times $\frac{1}{2}$ of A's; how much money has each?

PROBLEM 26.—Divide the number 60 into two parts, that shall be to each other as $\frac{1}{2}$ to $\frac{3}{4}$

PROBLEM 27.—The sum of two numbers is 140, and the larger is to the smaller as 1 to $\frac{5}{6}$; what are the numbers?

PROBLEM 28.—A and B together owe $207; B owes $\frac{11}{12}$ as much as A; how much does each owe?

PROBLEM 29.—I sold a horse for $\frac{1}{8}$ more than he cost me, receiving $270 for him; how much did he cost me?

PROBLEM 30.—What will $\frac{3}{4}$ of a barrel of flour cost at $11.28 per barrel?

PROBLEM 31.—What will $\frac{5}{6}$ of a bag of coffee weigh if a bag weighs 147 lbs?

PROBLEM 32.—What will $\frac{7}{8}$ of a pound of tea cost at $1.25 per pound?

PROBLEM 33.—What will $\frac{9}{10}$ of a cord of wood cost at $6.25 per cord?

PROBLEM 34.—What will $\frac{7}{10}$ of a hogshead of wine cost at $138.75 per hogshead?

Problem 35.—How much is $\frac{1}{3}$ and $\frac{1}{2}$ of $\frac{1}{3}$ of 15?

Problem 36.—A and B traded in company; A put in $\frac{2}{3}$ as much as B; they gained $750; what was each man's share?

Problem 37.—James says to John, give me $7.00 and I will have as much money as you. John says to James, give me $7.00 and I will have twice as much as you, Ans. 35 and 49.

Simply multiply the $7.00 by the numbers 5 and 7; and for all similar problems simply multiply the sum of money given, by the numbers 5 and 7.

Problem 38.—A says to B, give me 3\frac{1}{3}$ and I will have as much money as you. B says to A, give me 3\frac{1}{3}$ and 1 will have twice as much as you. How much money has each?

Problem 39.—Haight says to Booth, give me 1000 sheep and I will have as many as you. Booth says to Haight, give me 1000 and I will have twice as many as you. How much has each?

Problem 40.—Friedlander says to Reese, give me $500,000 and I will have as much as you. Reese says to Friedlander, give me $500,000 and I will have twice as much as you. How much has each?

Problem 41.—C says to D, give me 13.33\frac{1}{3}$ and I will have as much money as you. D says to C, give me 13.33$\frac{1}{3}$ and I will have twice as much money as you. How much has each?

Problem 42.—Greeley says to Grant, give me

50,000 votes and I will have as many as you. Grant says to Greeley, give me 50,000 votes and I will have twice as many as you; how many has each?

PROBLEM 43.—Two Hoodlums go into a saloon; one says to the other, give me as much money as I have, and I will spend two bits with you. They go into another saloon, and he says, give me as much money as I now have, and I will spend two bits with you. They went into the third saloon, and he made the same statement, and coming out of the third saloon he had nothing left. How much had he when he went into the first saloon? Ans., $1\frac{3}{4}$ bits. Simply $\frac{7}{8}$ of the sum spent in the first saloon is the answer.

PROBLEM 44.—A and B step into a hotel; A says to B, give me as much money as I have, and I will spend five dollars with you. They go into a second and third, A making the same statement; and coming out of the third, he had nothing left. How much had he when they went into the first hotel?

PROBLEM 45.—If 3 be the third of 6, what will the fourth of 20 be? Ans. $3\frac{1}{3}$.

SOLUTION.—The third of 6 is 2, if 3 be 2, 1 is $\frac{1}{3}$ of 2 or $\frac{2}{3}$, and 20 is 20 times $\frac{2}{3}$ or $\frac{40}{3}$, the $\frac{1}{4}$ of 20 is the $\frac{1}{4}$ of $\frac{40}{3}$ or $\frac{10}{3}$, $3\frac{1}{3}$ Ans.

PROBLEM 46.—If the third of 6 be 3, what will the fourth of 20 be? Ans. $7\frac{1}{2}$.

Solution.—If 2 be 3, 1 is ½ of 3, 1½ and 20 is 20 times 1½ or 30, ¼ of 20 is the ¼ of 30 or 7½ Ans.

UNITY FORM.

The circumference of a circle equals the diameter multiplied by 3.1416, the ratio of the circumference to the diameter.

The area of a circle equals the square of the radius multiplied by 3.1416.

The area of a circle equals one quarter of the diameter multiplied by the circumference.

The radius of a circle equals the circumference multiplied by 0.159155.

The radius of a circle equals the square root of the area multiplied by 0.56419.

The diameter of a circle equals the circumference multiplied by 0.31831.

The diameter of a circle equals the square root of the area multiplied by 1.12838.

The side of an inscribed equilateral triangle equals the diameter of the circle multiplied by 0.86.

The side of an inscribed square equals the diameter multiplied by 0.7071.

The circumference of a circle multiplied by 0.282 equals one side of a square of the same area.

The side of a square equals the diameter of a circle of the same area multiplied by 0.8862.

The area of a triangle equals the base multitiplied by one half of its altitude.

The area of an ellipse equals the product of both diameters and .7854.

The solidity of a sphere equals its surface multiplied by one-sixth of its diameter.

The surface equals the product of the diameter and circumference.

The surface of a sphere equals the square of the diameter multiplied by 3.1416.

The surface equals the square of the circumference multiplied by 0.3183.

The solidity of a sphere equals the cube of the diameter multiplied by 0.5236.

The diameter of a sphere equals the square root of the surface multiplied by 0.56419.

The square root of the surface of a sphere multiplied by 1.772454 equals the circumference.

The diameter of a sphere equals the cube root of its solidity multiplied by 1.2407.

The circumference of a sphere equals the cube root of its solidity multiplied by 3.8978.

The side of an inscribed cube equals the radius multiplied by 1.1547.

The solidity of a cone or pyramid equals the area of its base multiplied by one third of its altitude.

BUSINESS FORMS.

Note Negotiable by Delivery.

$100. Boston, June 23, 187—.

Four months after date, we promise to pay A. L., or bearer, one hundred dollars, value received.

A. B. & Co.

Note Negotiable by Indorsement.

$100. Boston, June 23, 187—.

Ninety days after date, I promise to pay A. L., or order, one hundred dollars, at the Suffolk Bank, Boston, value received. A. B.

Note not Negotiable.

$100. Boston, June 23, 187—.

Three months after date, I promise to pay A. L. one hundred dollars, value received. A. B.

Ordinary Inland Bill of Exchange, or Draft.

$100. Boston, June 23, 187—.

Three months* after date, pay to the order of G. W., One Hundred Dollars, value received, and charge the same to our account. C. D. & Co.

To E. F., Merchant, New York.

A Foreign Bill, or Set of Exchange.

$1000. Boston, June 23, 187—.

Sixty days* after sight of this First of Exchange, Second and Third of the same tenor and date unpaid,) pay to the order of C. D. & Co., in L——, the sum of One Thousand Dollars, value received, and charge the same to account as advised by A. B.

To. Mr. E. F., of C——.

* This admits of the following variations, according to circumstances: Instead of "threee months," or "sixty days," it may be "at sight," or at such a time "after sight," or at such a specific time, or on "demand."

RECEIPTS.

[A receipt is not conclusive, but only presumptive evidence, that the money therein mentioned has been paid; it may be denied, varied, or contradicted by parole or other evidence.]

Receipt in Full.

BOSTON, June 23, 187—. Received of A. B. twenty dollars, in full of all accounts. C. D.
$20.

Receipt on Account.

BOSTON, June 23, 187—. Received of A. B., ten dollars on account. C. D.
$10.

Receipt for Horse, with Warranty.

BOSTON, June 23, 187—. Received from A. B. two hundred dollars, for a bay gelding, warranted sound in every respect, quiet to ride and drive, and free from vice.
$200. C. D.

BORROWED MONEY.

$50. BOSTON, June 23 187—.
Borrowed and received of A. B. fifty dollars, which I promise to pay on demand, with interest. C. D.

DUE BILL.

BOSTON, June 23, 187—. Due, on demand, to A.B., one hundred dollars, value received. N. O.

ORDERS.

Mr. A. B. BOSTON, June 23, 187—.
Please to pay A. L. ten dollars, in merchandise, and charge the same to my account. N .O.

Mr. A. B. BOSTON, June 23, 187—.
Please pay C. D., or bearer, ten dollars, and charge the same to my account. E. F.

POPULATION OF THE WORLD.

The latest and most reliable statistics on the subject estimate the population of the World at 1,375,000,000, which is thus divided among the different continents:

Europe	293,000,000
Asia	805,400,000
America	81,400,000
Africa	191,000,000
Australia, etc	4,200,000
Total	1,375,000,000

Reckoning the average deaths as about one in every forty inhabitants, 32,000,000 die in a year; 87,671 in a day; 3,633 in an hour; and 61 in a minute. Thus, one human being dies on an average every second, and more than one is born.

The entire population is thus divided in point of religion:

Christians—	Protestants	100,835,000
	Roman Catholics	195,460,200
	Eastern Church	81,478,000
		377,775,200
Buddhists		360,000,000
Other Asiatic Creeds		260,000,000
Pagans		200,000,000
Mohammedans		165,000,000
Jews		7,000,000

PRESIDENTS OF THE UNITED STATES,

FROM THE ADOPTION OF THE CONSTITUTION.

NO.	NAMES.	RESIDENCE.	BORN.	INSTALLED.
1	George Washington.......	Virginia.........	1732	1789
2	George Washington.......	....	....	1793
3	John Adams................	Massachusetts	1735	1797
4	Thomas Jefferson.........	Virginia	1743	1801
5	Thomas Jefferson	....	....	1805
6	James Madison............	Virginia.........	1751	1809
7	James Madison............	....	....	1813
8	James Monroe.............	Virginia.........	1758	1817
9	James Monroe	....	....	1821
10	John Q. Adams	Massachusetts	1767	1825
11	Andrew Jackson	Tennessee	1767	1829
12	Andrew Jackson	....	....	1833
13	Martin Van Buren.........	New York......	1782	1837
14	Wm. H. Harrison*........	Ohio...............	1773	1841
	John Tyler..................	Virginia.........	1790	1841
15	James K. Polk..............	Tennessee	1795	1845
16	Zachary Taylor.*...........	Louisiana	1784	1849
	Millard Fillmore...........	New York......	1800	1850
17	Franklin Pierce............	N. Hampshire	1804	1853
18	James Buchanan..........	Pennsylvania	1791	1857
19	Abraham Lincoln.........	Illinois...........	1809	1861
20	Abraham Lincoln*........	....	....	1865
	Andrew Johnson..........	Tennessee	1808	1865
21	Ulysses S. Grant	Illinois............	1822	1869
22	Ulysses S. Grant	....	....	1873
23	Rutherford B. Hayes......	Ohio	1822	1877
24	James A. Garfield*.........	Ohio	1831	1881
	C. A. Arthur..................	Vermont..........	1830	1881

*Died during term of office.

The author of this book graduated at same college as C. A. Arthur.

TABLE,

SHOWING DIFFERENCE OF TIME IN VARIOUS CITIES WHEN IT IS 12 NOON AT NEW YORK.

New York	12.00 M.	Boston	12.12 P.M.
Buffalo	11.40 A.M.	Quebec	12.12
Cincinnati	11.18	Portland	12.15
Chicago	11.07	London	4.55
St. Louis	10.55	Paris	5.05
San Franc'o.	8.45	Rome	5.45
New Orleans	10.56	Constan'ople	6.41
Washington	11.48	Vienna	6.00
Charleston	11.36	St. Petersb'g	6.57
Havana	11.25	Pekin	12.40 A.M.

WEIGHTS AND MEASURES.

TROY WEIGHT.

By this weight gold, silver, platina and precious stones (except diamonds) are estimated.

20 mites...1 grain.	20 pennyw'ts..1 ounce.
20 grains..1 pennyw't.	12 ounces......1 pound.

Pure gold is 24 carats fine. The U. S. standard for gold coin is nine-tenths pure gold.

The term carat is also applied to a weight of 3½ grains troy, used in weighing diamonds; it is divided into 4 parts called *grains*; 3½ grains troy are thus equal to 4 grains diamond weight.

APOTHECARIES' WEIGHT.

The pound and ounce of this weight are the same as the pound and ounce troy, but differently divided.

20 grains troy.........1 scruple......℈
3 scruples............1 drachm......ʒ
8 drachms...........1 ounce troy...℥
12 ounces............1 pound troy...℔

Wholesale druggists sell their goods by

AVOIRDUPOIS WEIGHT.

By this weight all goods are sold except those named under troy weight.

16 drachms	1 ounce.
16 ounces	1 pound.
25 pounds	1 quarter.
4 quarters	1 hundred weight.
20 hundred weight	1 ton.

APOTHECARIES' FLUID MEASURE.

60 minims	1 fluid drachm	*f*ʒ.
8 fluid drachms	1 ounce (troy.)	*f*℥.
16 ounces (troy)	1 pint.	
8 pints	1 gallon.	

LIQUID, OR WINE MEASURE.

4 gills (*gil.*)	make	1 pint	*pt.*
2 pt.	"	1 quart	*qt.*
4 qt.	"	1 gallon	*gal.*
$31\frac{1}{2}$ gal	"	1 barrel	*bbl.*
2 bbl.	"	1 hogshead	*hhd.*

The U. S. wine gallon contains 231 cubic inches.

DRY MEASURE.

4 gills	1 pint.
2 pints	1 quart.
4 quarts	1 gallon.
2 gallons	1 peck.
4 pecks, or 8 gals.	1 bush.
36 bushels	1 chaldron.

CLOTH HEASURE.

$2\frac{1}{4}$ inches	1 nail.
4 nails	1 quarter of a yard.
4 quarters	1 yard.
5 quarters	1 ell, English.

LONG MEASURE.

12 inches	1 foot.
3 feet	1 yard.
5½ yards	1 rod, pole or perch.
40 poles	1 furlong.
8 furlongs, or 1760 yards	1 mile.

MISCELLANEOUS LENGTHS.

12 inches (*in.*)	make	1 foot	*ft.*
3 ft.	"	1 yard	*yd.*
5½ yd.	"	1 rod or pole	*rd.* or *p.*
40 rd.	"	1 furlong	*fur.*
8 fur.	"	1 statute mile	*mi.*

SURVEYORS' LONG MEASURE.

7.92 inches (*in.*)	make	1 link	*l.*
25 l.	"	1 rod or pole	*rd.* or *p.*
4 rd., or 66 ft.,	"	1 chain	*ch.*
80 ch.	"	1 mile	*mi.*

SQUARE MEASURE.

144 square inches	1 square foot.
9 square feet	1 square yard.
30¼ square yards	1 square rod or perch.
40 square rods	1 rood.
4 roods	1 acre.
640 acres	1 square mile.

U. S. GOVERNMENT LAND MEASURE.

A township—36 sections, each a mile square.

A section—640 acres.

A quarter section, half a mile square—160 acres.

An eighth section, half a mile long, north and south, and a quarter of a mile wide—80 acres.

A sixteenth section, a quarter of a mile square—40 acres.

The sections are all numbered 1to 36, commencing at the northeast corner, thus :

6	5	4	3	2	NW | NE SW | SE
7	8	9	10	11	12
18	17	16*	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

* School Section.

The sections are divided into quarters, which are named by the cardinal points, as in section 1. The quarters are divided in the same way. The description of a forty acre lot would read : The south half of the west half of the southwest quarter of section 1 in township 24, north of range 7 west, or as the case might be; and sometimes will fall short and sometimes overrun the number of acres it is supposed to contain.

NAUTICAL MEASURE

6 feet.....................1 fathom.
120 feet.....................1 cable in length.
110 fathoms, or 660 ft..1 furlong.
6075$\frac{4}{5}$ feet....................1 nautical mile.
3 nautical miles........1 league.
20 leagues, or 60geo.m.1 degree.
360 degrees ....The earth's circumference.
=24,855$\frac{1}{2}$ miles, nearly.

The nautical mile is 795⅘ feet longer than the common mile.

CUBIC MEASURE

1728 cubic inches..............1 cubic foot.
27 cubic feet..................1 cubic yard.
16 cubic feet..................1 cord ft., or ft of wood.
8 cord ft., or 128 cub ft...1 cord.
40 ft. or round, or 50 ft. of hewn timber. } 1 ton.
40 cubic feet..................1 ton of shipping.

CIRCULAR MEASURE.

60 seconds..............................1 minute.
60 minutes..............................1 degree.
60 degrees..............................1 sign.
90 degrees..............................1 quadrant.
360 degrees..............................1 circumference.

MEASURE OF TIME.

60 seconds..........................1 minute.
60 minutes..........................1 hour.
24 hours..........................1 day.
7 days..........................1 week.
28 days..........................1 lunar month.
28, 29, 30, or 31 days............1 calendar month.
12 calendar months...............1 year.
365 days..........................1 common year.
366 days..........................1 leap year.
365¼ days..........................1 Julian.
365 d. 5 h. 48 m. 49 s...............1 solar year.
365 d. 6 h. 9 m. 12 s...............1 siderial year.

The densities of most fluids and solid bodies have been compared with that of water; and the number expressing the amount that one cubic inch of a body is heavier or lighter than the same amount of water, is called the density or the *specific gravity* of the body. The following are the specific gravities of a few well-known bodies:

Substances.	Specific Gravity.	Substances	Specific Gravity.
Cork	0.24	Quartz	2.6
Poplar wood	0.38	Basalt	2.66
Lime-tree wood	0.439	Bottle-glass	2.60
Pine wood	0.555	Granite	2.80
Nut-tree wood	0.677	Diamond	3.52
Ether	0.713	Heavy spar	4.426
Alcohol	0.793	Chromium	5.900
Oil of Turpentine	0.872	Antimony	6.712
Poppy oil	0.929	Zinc	7.037
Ice	0.916	Iron (wrought)	7.788
Water	1.000	Steel	7.816
Sea-water	1.026	Copper (wrought)	8.878
Milk	1.030	Bismuth	9.820
Oak wood	1.170	Silver	10.474
Phosphorus	1.770	Lead	11.852
Sulphuric acid	1.848	Mercury	13.598
Ivory	1.917	Gold	19.325
Sulphur	2.03	Platinum	22.100

The advantage of a knowledge of the above is easily proved. For instance, as every substance invariably possesses a uniform density under equal conditions, we arrive at one of the most important means of recognizing a body. In purchasing pure silver, each cubic inch should weigh 5.237 ounces. Should its density be less, the silver may be assumed to contain copper, if it be greater, lead my be present.

ALPHABET OF NUMBERS.

The new alphabet of numbers is founded in the constitution of nature. The first character $\frac{1}{1}$, one one, is the alpha in every investigation; the one below the dividing line representing the unity, and the one above the line the unit of the denomination. Thus the specific gravity of gold is $\frac{19.325}{1}$ times that of water, the expression below the line representing the unity of measure, or the water, and what is above the line the specific gravity of gold when compared with water. The cause of any thing that we can examine contains two elements, the positive and the negative. The positive and negative forces unite in producing all forms in nature, animate and inanimate. The mineral, the vegetable, the animal and the human have the positive and negative, or the masculine and feminine. We have in arithmetic only two fundamental rules, the *plus* and the *minus*. We have in the first numerical character the two expressions, representing the unity and the unit, $\frac{1}{1}$, in the second character $\frac{2}{1}$, in the third $\frac{3}{1}$, &c., to the last which is represented by $\frac{0}{1}$. Hence we have ten characters because there are ten ones, $\frac{10}{1}$, in the base of our system of notation. The unity, is understood if not expressed in every whole number, and appears in the denominator of the fraction divided into equal parts; thus $\frac{10}{1}$ are contained in one, $\frac{1}{10}$ times, found by

inverting the ten, the unity taking the place of the ten, and the ten the place of the unity; now the ten represents the unity divided into ten equal parts, and the unity represents one of the parts. Hence inverting any number demonstrates the number of times that number is contained in one, for that reason we invert the divisor in division of fractions, then multiply by the number of ones in the dividend to find the quotient. For a similar reason we invert the rate, in my rule of interest, to find how many times it is contained in one, or the time it takes a dollar to earn a cent.

$\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$ and $\frac{1}{0}$,

Represent the characters of the alphabet of our system of notation inverted, and the number of times each character is contained in one. One is contained in one one times, two one-half times, three one-third times, four one-fourth times, &c., to zero, which is contained an infinite number of times.

We find that nearly every rule is reduced to the form of the first character, thus: Gold and currency, 100, the number of cents in a dollar represents the numerator, in the statement, and the price of the gold or currency, as the case may be, the denominator. In measuring land $\frac{5}{8}$ of the number of hundred rods equals the number of acres; measuring grain $\frac{8}{10}$ of the number of cubic feet equals the number of bushels, by adding $4\frac{1}{2}$ bushels for every thousand so found.

MEASURING GRAIN.

Thus a bin 18 by 20 feet and 6 feet deep contains $18 \times 20 \times 6$, or 2160 cubic feet; now removing the decimal point one place to the left we have 216, and multiplying that result by 8, thus $216 \times 8 = 1728$, the number of bushels in the bin, lacking $4\frac{1}{2}$ for each thousand. To make the correction remove the point three places, and multiply by $4\frac{1}{2}$.

$$\begin{array}{r} \text{Thus,}\quad 1.728 \\ 4\frac{1}{2} \\ \hline 6.912 \\ .864 \\ \hline 7.776 \end{array}$$

$$1728 + 7.776 = 1735.776$$

the number of bushels in the bin.

The reason of each step becomes evident by a moments observation. The ratio of 1728, the number of cubic inches in a cubic foot, to 2150.4 cubic inches in a bushel is $\frac{8}{10}$ nearly, we add $4\frac{1}{2}$ for each thousand for the correct result. Hence the rule for all examples: *Remove the decimal point one place to the left in the number of cubic feet, and multiply by 8, and add $4\frac{1}{2}$ for each thousand.*

A bin 10 by 10 and 5 feet deep, contains 500 cubic feet. Removing the point one place we have 50, and multiplying by 8 gives 400, and making the correction we have 401.8, the number of bushels in the bin. This rule is so plain that any one can put a measurnig reed in bin or box and grain is measured and weighed at sight.

MEASURING LAND.

The rule of measuring land is somewhat similar. 160 rods make an acre. Now removing the decimal point two places to the left, to reduce it to units of a hundred, and dividing by 8 and multiplying by 5, we get 1, thus: $1.60 \div 8 \times 5 = 1$. $\frac{5}{8}$ of the number of hundred rods is the number of acres. Hence the rule: *Remove the decimal point two places to the left in the number of rods, and divide by 8 and multiply by 5 in all examples.* Thus a field 2400 rods long and 200 wide, removing the point two places to the left in the first factor, we get 24, and dividing by 8 and multiplying by 5, gives 15, which multiplied by 200, the other factor, gives 3000, the number of acres in the field. Forty chains make an acre. Hence: *Remove the decimal point one place to the left in the number of chains and divide by 4. Thus 40, removing the point one place, gives 4, and 4, divided by 4, gives 1.*

148 chains, how many acres? Removing the point one place to the left we have 14.8, and dividing that result by 4 gives 3.7 acres

EXAMPLES IN MEASURING LAND.

N. B. The unit of width is the rod in these examples.

1.—How many acres in a field 320 chains long and 131 rods wide?

SOLUTION.—Removing the point one place in 320, gives 32, and dividing 32 by 4 we get 8, and

multiplying 131 by 8 gives 1048 the numbers of acres in the field.

2.—How many acres in a field 320 rods long and 131 wide?

Solution.—Removing the point two places to the left in 320, and dividing by 8 and multiplying by 5 gives 2, and 131 multiplied by 2 gives 262 the number of acres in the field.

The rules just illustrated reach all examples without any change in their application.

3.—How many acres in 2400 rods?

4.—How many acres in 3751 chains?

5.—How many acres in 640 chains multiplied by 356?

6.—How many acres in 6407 rods multiplied by 791?

7.—How many acres in 720 rods multiplied by one half of 300?

8.—How many acres in 280 chains multiplied by one fourth of 1200?

9.—How many acres in a field 3200 rods long and 2171 wide?

10.—How many acres in a triangular field, the base 320 rods and altitude 200?

The area of any triangle is the base multiplied by half the altitude.

11.—How many acres in a triangle, the base 80 rods and altitude 30?

12.—How many acres in a triangle, the base being 80 chains and the altitude 60?

The area of any circle is the circumference multiplied by one quarter of its diameter.

13.—*How many acres in a circle* of 600 chains diameter? the circumference being 3.1416 times the diameter.

14.—How many acres in 680 rods multiplied by 900?

15.—How many acres in 347 chains multiplied by 671?

16.—How many acres in 3280 rods multiplied by 75?

17.—How many acres in 400 chains multiplied by 131?

18.—How many acres in 4000 chains multiplied by 144?

19.—How many acres in 980 rods multiplied by 640?

20.—How many acres in 12 chains multiplied by 16?

21.—How many acres in 6 chains multiplied by 7?

EXAMPLES IN MEASURING GRAIN.

1.—How many bushels in a bin 6 by 18 and 5 feet deep?

Rule and Solution.—*Eight tenths of the number of cubic feet is the number of bushels.* Hence, removing the point one place to the left in the number 5, gives five tenths or one half, and 8 times ½

is four; and $4 \times 6 \times 18 = 432$ the number of bushels, nearly.

2.—How many bushels in a bin 12 by 20, and 7 feet deep?

3.—How many bushels in a bin 10 by 10 and 5 feet deep?

4.—How many bushels in 2130 cubic feet?

5.—How many bushels in 1728 cubie feet?

6.—Why is eight tenths of the number of cubic feet the number of bushels nearly?

7.—Why does removing the decimal point one place to the left in any number divide it by ten?

8.—How many bushels in 1600 cubic feet?

9.—How many bushels in 10000 cubic feet?

10.—How do you make the correction after removing the point one place and multiplying by 8? Simply remove the point three places to tell the number of thousand bushels and multiply by 4½.

BUYING AND SELLING BY THE TON.

RULE AND REASON OF THE RULE.—*Remove the decimal point three places to the left in the number of pounds, and multiply by half the price per ton.*

Removing the point three places to the left to reduce to units of a thousand, and dividing the price by two gives the cost of one thousand pounds, and multiplying the number of thousand by the cost of one thousand gives the answer in all cases when two thousand pounds make a ton.

1.—What is the cost of 3000 pounds of hay at \$14.00 per ton.

SOLUTION.—Removing the decimal point three places to the left in three thousand gives 3, and since two thousand pounds or one ton costs \$14.00, one thousand costs one half of 14, or seven, and $3 \times 7 = 21$ the number of dollars.

2.—What is the cost of 6721 pounds at \$14.00 per ton?

Removing the point three places to the left we have 6.721, and multiplying by one half of 14, we have

$$\begin{array}{r} 6.721 \\ 7 \\ \hline \$47.047 \end{array}$$

for the result.

3.—What will 100000 pounds cost at \$16.00 per ton?

Removing the point three places to the left and multiplying by 8 the half of 16, gives \$800.00 ans.

4.—What is the cost of 1347 pounds at \$18.50 per ton? Remove the point three places to the left and multiply by $9\frac{1}{4}$.

5.—What is the cost of 17891 pounds at \$22.25 per ton? Remove the point three places to the left in the number of pounds and multiply by $11\frac{1}{8}$.

6.—What is the cost of 39891.5 pounds at \$50.25 per ton? Removing the point three places in the number of pounds and multiplying by $25\frac{1}{8}$ gives the cost. To multiply by 25 divide by four and call the result hundreds, and to multiply by $\frac{1}{8}$ divide by 8.

When the price is \$20.00 per ton, simply remove the decimal point two places to the left in the

number of pounds, because removing three places and multiplying by 10 the half of 20 is equivalent to removing it two places.

7.—What is the cost of 1371 pounds at $20.00 per ton?

$13.71, simply removing the point two places in the numbers of pounds.

8.—What is the cost of 47.5 pounds at $20.00 per ton?

BUYING AND SELLING BY THE HUNDRED.

RULE.—*Remove the point two places to the left and multiply by the price of one hundred in all cases.*

1.—What is the cost of 385 cigars at $3.50 per hundred?

Remove the decimal point two places to the left in the number to reduce it to units of a hundred, and multiply by the price of one hundred.

2.—What is the cost of 387 pounds of flour at $2.50 per hundred?

Remove the point two places to the left in the number of pounds, and multiply by $2\frac{1}{2}$, the price of one hundred pounds.

3.—What is the cost of 860 feet of lumber at $7.00 per hundred?

Removing the point two places to the left and multiplying by the price per hundred will give the cost.

4.—What is the cost of 13341 fence rails at $1.25 per hundred?

Remove the decimal point two places to the left in the number and multiply by $1\frac{1}{4}$.

5.—What is the cost of 27.5 pounds of corn meal at $2.25 per hundred?

Remove the point two places to the left on the number of pounds and multiply by $2\frac{1}{4}$.

6.—What will 47.75 pounds of flour cost at $3.12½ per hundred?

Remove the decimal point two places to the left in the number of pounds and multiply by $3\frac{1}{8}$.

BUYING AND SELLING BY THE THOUSAND.

RULE FOR ALL EXAMPLES.—*Remove the decimal point three places to the left in any number to reduce to units of a thousand, and multiply by the price of one thousand.*

1.—What is the cost of 2321 feet of lumber, at $8.00 per thousand?

Removing the decimal point three places to the left we have 2.321, and multiplying by 8, the cost of one thousand, gives $18.568 for the answer.

2.—What is the cost of 17891 feet of lumber, at $12.00 per thousand?

Remove the point three places to the left and multiply by 12.

3.—What is the cost of 10000 feet of lumber, at $13.50 per thousand?

Remove the point three places to the left and multiply by 13½.

4.—What is the cost of 13791 feet of lumber, at $12.25 per thousand?

Remove the point three places to the left and multiply by 12¼.

5.—What is the cost of 15621 shingles, at $11.25 per thousand?

Remove the decimal point three places to the left and multiply by 11¼.

6.—What is the cost of 321575 feet of lumber, at $8.50 per thousand?

Remove the decimal point three places to the left and multiply by 8½.

CALCULATING INTEREST.

My method is explained in the fore part of this work. Here we add some illustrations to aid the student in comprehending thoroughly the full application of the rule.

Explanation of Rule.—Inverting the rate, gives the time it takes a dollar to earn a cent, and removing the decimal point two places to the left, gives the interest of any sum of money for the first period of time; annexing a cipher to that period of time, gives the time it takes a dollar to earn ten cents, and removing the decimal point

one place to the left, gives the interest of any sum of money for the second period of time; prefixing a point, or dividing the first period of time found by inverting the rate by ten, gives the time it takes a dollar to earn a mill or the tenth of a cent, and removing the point three places to the left, gives the interest of any sum of money for the third period of time; removing the decimal point one place to the left on the third period of time, gives a fourth period of time, in which a dollar earns one hundredth part of a cent, and removing the decimal point four places to the left in any sum, gives the interest for the fourth period of time; removing the decimal point one place to the left in fourth period of time, gives a fifth period of time, in which a dollar earns one thousandth part of a cent, and removing the decimal point five places to the left, gives the interest of any sum of money for the fifth period of time; removing the decimal point one place to the left in the fifth period of time, gives the sixth period of time, in which a dollar earns one ten-thousandth part of a cent, and removing the decimal point six places to the left, gives the interest of any amount of money for the sixth period of time; removing the decimal point one place to the left in the sixth period of time, gives the seventh period of time, in which a dollar earns one hundred-thousandth part of a cent, and removing the decimal point seven places to the left, gives the interest of any sum of money for the seventh period of time; removing the dec-

imal point one place to the left in the seventh period of time, gives the eighth period of time, in which a dollar earns one-millionth part of a cent, and removing the decimal point eight places to the left, gives the interest of any sum of money for the eighth period of time; removing the decimal point one place to the right in the second period of time, in which a dollar earns ten cents, and it gives a ninth period of time, in which a dollar earns one hundred cents and the principal equals the interest, and the decimal point remains unchanged; annexing a cipher to the ninth period of time, gives the tenth period of time, in which one dollar earns ten, and removing the decimal point one place to the right in any sum of money, gives the interest for the tenth period of time, &c.

Increasing and diminishing results, all business periods of time are quickly and conveniently reached. Illustration of the fore-going explanation, which applies to all rates and all sums of money.

Per annum 12 per cent.

Remove the decimal point two places for one month, three places for three days, four places for three-tenths of a day, five places for three-hundredths of a day, six places for three-thousandths of a day, seven places for three ten-thousandths of a day, and eight places for three one-hundred-thousandths of a day; for ten months remove the decimal point one place to the left, one hundred months the point remains unchanged, and the

principal is the interest, one thousand months remove the point one place to the right. Thus any rate per cent is handled, and two, three or four periods of time is quite sufficient for business purposes. Per annum 10 per cent., 9, 8, 7, 6, 5, 4, and 3 per cent. according to the rule we have the several periods of time 36 days, 1 year and 10 years for ten per cent; for 9 per cent., 4 days, 40 days, 400 days, 4000 days; for eight per cent, 4.5 days, 45 days, 15 months, 150 months; for seven per cent., 5.2 days, 52 days, 520 days, 5200 days; for six per cent., 6 days, 2 months, 20 months, 200 months; for five per cent., 7.2 days, 72 days, 2 years, 20 years; for four per cent., 9 days, 3 months, 30 months, 300 months; for three per cent. the rule gives 12 days, 4 months, 40 months, 400 months.

	10	pr. ct.,	3.6	ds.,	36 ds.,	1 yr.,	10 yr.
	9	"	4	"	40 "	400 ds.,	4000 ds.
	8	"	4.5	"	45 "	15 mo.,	150 mo.
	7	"	5.2	"	52 "	520 ds.,	5200 ds.
	6	"	6	"	2 mo.,	20 mo.,	200 mo.
	5	"	7.2	"	72 ds.,	2 yr.,	20 yr.
	4	"	9	"	3 mo.,	30 mo.,	300 mo.
	3	"	12	"	4 "	40 "	400 "
	$4\frac{1}{2}$	"	8	"	80 ds.,	800 ds.,	8000 ds.
	$3\frac{1}{3}$	"	10.8	"	108 "	3 yr.,	30 yr.
	$4\frac{2}{7}$	"	8.4	"	84 "	$2\frac{1}{3}$ "	$23\frac{1}{3}$ "
	$8\frac{4}{7}$	"	4.2	"	42 "	14 mo.,	140 mo.
Per mo.	$1\frac{1}{2}$	"	2	"	20 "	200 ds.,	2000 ds.
"	2	"	1.5	"	15 "	5 mo.,	50 mo.
"	$2\frac{1}{2}$	"	1.2	"	12 "	4 "	40 "
"	1	"	3	"	1 mo.,	10 "	100 "
"	$\frac{3}{4}$	"	4	"	40 ds.,	400 ds.,	4000 ds.
"	$1\frac{1}{4}$	"	2.4	"	24 "	8 mo.,	80 mo.

Illustration,

$ 8	7	6	0.	75	
	3	4	1.	80	
96	9	4	3.	78	
50	5	0	0.	25	
	6	0	0.	00	
7	0	0	0.	50	&c. &c.

All examples are performed without making a figure. The periods of time are found in a moment by my rule on page seventeen of this work, where it is thoroughly explained and the reason given for each operation. By the same rule we can remove the point on the time, or number of days, to find the interest; because inverting the rate gives the time it takes a dollar to earn a cent, and annexing a cipher to that time gives the time it takes a dollar to earn ten cents, &c. Hence, at 8 per cent. per annum one dollar earns one cent in 45 days, hence $45 earn one cent in one day, and one hundredth part of the number of days equals the interest; and 450 dollars earn ten cents in one day, and one tenth of the number of days equals the interest. $4.50 at 8 per cent. per annum in one day earns a mill, hence one thousandth part of the number of days equals the interest, &c. &c. Inverting the rate finds the number of dollars it takes to earn one cent in one day at the given rate in all rates; annexing a cipher to that number of dollars, gives the number of dollars it takes to earn ten cents in one day; removing the decimal point one place to the left in the same number of dollars gives the number of dollars it takes at the given

rate to earn one mill in one day, and removing the decimal point three places to the left in the number of days gives the interest of the sum of money for any number of days that you can write; thus:

pr. ann.	8	pr. ct.,	$4.50	$45.00	$450.00	$4500.00
"	9	"	4.00	40.00	400.00	4000.00
"	12	"	3.00	30.00	300.00	3000.00
"	6	"	6.00	60.00	600.00	6000.00
"	3	"	12.00	120.00	1200.00	12000.00
"	$7\frac{1}{2}$	"	4.80	48.00	480.00	4800.00
"	$4\frac{2}{7}$	"	8.40	84.00	840.00	8400.00
per mo.	$1\frac{1}{2}$	"	2.00	20.00	200.00	2000.00
"	$1\frac{1}{4}$	"	2.40	24.00	240.00	2400.00
"	2	"	1.50	15.00	150.00	1500.00
"	3	"	1.00	10.00	100.00	1000.00

Now the interest is found by removing the decimal point on the number of days, and prefixing the sign of dollars.

1	2	3	6
	9	8	7
	4	3	3
	1	1	1
		5	8
	1	4	4
1	3	8	9
		1	5
		1	0
			1

TO MARK GOODS BOUGHT BY THE DOZEN.

RULE.—*Removing the decimal point one place to the left in the price per dozen to gain 20 per cent.; increase or diminish to suit the required rate.*

Thus: 12 hats at $12.00 per dozen; removing the point one place to the left in the price gives $1.20, the selling price per hat to make 20 per cent.; at $14.00, $1.40; at $18.00, $1.80, &c. for all prices that may occur. To make 80 per cent. remove the point and add one half itself.

Increasing and diminishing from the established base all rates are quickly found.

SPECIAL RULES FOR MULTIPLYING

WHOLE AND FRACTIONAL NUMBERS.

21	22	23	24	25
29	28	27	26	25
609	616	621	624	625

RULE.—Add one to the two and multiply by two, and annex the product of the units. This rule will apply in all cases when the tens are equal and the sum of the units make ten, both in whole and fractional numbers.

$$\begin{array}{r} 87 \\ 83 \\ \hline 7221 \end{array} \qquad \begin{array}{r} 86 \\ 84 \\ \hline 7224 \end{array} \qquad \begin{array}{r} 88 \\ 82 \\ \hline 7216 \end{array} \qquad \begin{array}{r} 85 \\ 85 \\ \hline 7225 \end{array}$$

$$76 \times 74 = 5624$$
$$73 \times 77 = 5621$$
$$78 \times 72 = 5616$$
$$7\tfrac{8}{10} \times 7\tfrac{2}{10} = 56.16$$
$$7.4 \times 7.6 = 56.24$$
$$7.5 \times 7.5 = 56.25$$
$$4\tfrac{2}{3} \times 4\tfrac{1}{3} = 20\tfrac{2}{9}$$
$$4\tfrac{1}{7} \times 4\tfrac{6}{7} = 20\tfrac{6}{49}$$
$$8\tfrac{1}{2} \times 8\tfrac{1}{2} = 72\tfrac{1}{4}$$
$$8\tfrac{3}{4} \times 8\tfrac{1}{4} = 72\tfrac{3}{16}$$
$$9\tfrac{2}{3} \times 9\tfrac{1}{3} = 90\tfrac{2}{9}$$
$$7\tfrac{7}{8} \times 7\tfrac{1}{8} = 56\tfrac{7}{64}$$

&c. for all similar cases.

When numbers occur not similar to these, apply the general rule.

Rule.—Multiply the units by the units, and the units by the tens adding the results mentally, and the tens by the tens. Thus:

$$\begin{array}{r} 32 \\ 43 \\ \hline 1376 \end{array}$$

3 times 2 is 6; 3 times 3 is 9, 2 times 4 is 8, 8 and 9 are 17, and 4 times 3 is 12 and one added makes 13.

Abgekürzte neue Rechnungsmethode

des

J. A. Henderson,

und um dieselbe leicht begreiflich zu machen.

Die Maßregel derselben sind folgender Art:

1. Das numerirte Alphabet kennen zu lernen, wie folgt:

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \frac{6}{1}, \frac{7}{1}, \frac{8}{1}, \frac{9}{1}, \frac{0}{1}$$

2. Das stipulirte Interesse nach Methode umzudrehen, wie folgt:

3 pr. Ct. per Monat oder $\frac{3}{1}$, umgedreht $\frac{1}{3}$ Monat = 10 Tage.

3. Für das zehnfache eine Null oder Zero zu anexiren, wie folgt:

$\frac{1}{3}$ Monat × bei 10 oder $\frac{10}{3}$ Monat = 100 Tage.

4. Für den zehnten Theil eine Null (Zero) zu präfixiren, wie folgt:

$\frac{1}{3}$ Monat ÷ bei 10 oder $\frac{01}{3}$ Monat = 1 Tag.

Die Gründe sind folgende:

Das Umdrehen der Interessen demonstrirt immer die Zeit, wenn ein Thaler ($ 1.00) einen Cent (1 ct.) macht.

Das anexiren einer Null (0) verzehnfacht die Zeit, gleich dies:

1 Cent zu 10 Cents.

Das präfixiren eines Punktes oder Null (0) verkürzt die Zeit zehnfach, daher auch der Cent als eine Mille oder $\frac{1}{10}$ Cent gerechnet werden muß.

Beispiel:

3 pr. Ct. per Monat [ist×]

	3		nach dem Alphabet $\frac{3}{1}$
	1	0	
	3		umgedreht macht es ⅓ Monat oder 10 Tage.

	1 Tag.	10 Tage.	100 Tage.	
$	15	0	0	0.00
	9	0	0	0.00
	1	2	0	0.75
	1	3	6	0.25

Erklärung.

Jedermann, der daher die oben gelieferten Maßregeln gründlich gelernt hat, wird leicht einsehen, mit welcher Leichtigkeit diese Methode alle

Rechnungen lößt, indem sie das gewöhnlich Schwierige durch Decimal angreift.

Hat man daher ausgefunden wenn $ 1.00 einen Cent macht, so wird Jedermann klar einsehen, daß er nur die Zahl der Thaler als Cents zu betrachten hat, um die Lösung derselben zu finden.

Sollte man nun wünschen auszufinden die Interessen von einer Summe zu 3 per Cent per Monat für 133 Tage, so gibt die Kenntniß dieser Methode ebenfalls eine leichte Lösung.

[Zum Beispiel] Die Interessen von $ 1.00 zu 3 per Ct. per Monat:

3 per Ct. per Monat ist = ⅓ Monat = 10 Tage (und macht 1 Cent in 10 Tagen) folglich für 133 Tage:

für	100	Tage	das	Zehnfache	v.	10	Tagen	od.	1	Ct.	=	10
„	30	„	„	Dreifache	„	10	„	„	1	„	=	3
„	3	„	3mal d. 10ten Theil		v.	10	T.	„	1	„	=	0.3
											Cents	13.3

Per Jahr 12 per Cent.

Versetze den Decimalpunkt um zwei Stellen nach links für einen Monat, drei Stellen für drei Tage, vier Stellen für drei Zehntel eines Tages, fünf Stellen für drei Einhundertstel eines Tages, sechs Stellen für drei Eintausendstel eines Tages, sieben Stellen für drei Zehntausendstel eines Tages, und acht Stellen für drei Einhunderttausendstel eines Tages; für zehn Monate rücke den Decimalpunkt um eine Stelle zur Linken; für hundert Monate bleibt der Punkt unverändert, denn die Interessen sind dem Kapital gleich; für tausend Monate rücke den Punkt um eine Stelle zur Rechten.

So wird jeder Zinsfuß berechnet, und zwei, drei oder vier Zeitperioden sind genügend für Geschäftszwecke. Per Jahr $\frac{10}{1}$ per Cent, 9, 8, 7, 6, 5, 4 und 3 per Cent, haben wir gemäß der Regel die verschiedenen Zeitperioden: für 10 per Cent 3.6 Tage, 36 Tage, 1 Jahr und 10 Jahre; für 9 per Cent 4 Tage, 40 Tage, 400 Tage, 4000 Tage; für 8 per Cent 4.5 Tage, 45 Tage, 15 Monate, 150 Monate; für 7 per Cent 5.2 Tage, 52 Tage, 520 Tage, 5200 Tage; für 6 per Cent 6 Tage, 2 Monate, 20 Monate, 200 Monate; für 5 per Cent 7.2 Tage, 72 Tage, 2 Jahre, 20 Jahre; für 4 per Cent 9 Tage, 3 Monate, 30 Monate, 300 Monate; und für 3 per Cent gibt die Regel 12 Tage, 4 Monate, 40 Monate, 400 Monate.

Jährlich	10	pr. Ct.	3.6 Tage,	36 Tage,	1 Jahr,	10 Jahre.
"	9	"	4 Tage,	40 Tage,	400 T.,	4000 T.
"	8	"	4.5 Tage,	45 Tage,	15 M.,	150 M.
"	7	"	5.2 Tage,	52 Tage,	520 T.,	5200 T.
"	6	"	6 Tage,	2 Mon.,	20 M.,	200 M.
"	5	"	7.2 Tage,	72 Tage,	2 Jahre,	20 Jahre.
"	4	"	9 Tage,	3 Mon.,	30 M.,	300 M.
"	3	"	12 Tage,	4 Mon.,	40 M.,	400 M.
"	4½	"	8 Tage,	80 Tage,	800 T.,	8000 T.
"	3⅓	"	10.8 Tage,	108 T.,	3 Jahre,	30 Jahre.
"	$4\frac{2}{7}$	"	8.4 Tage,	84 Tage,	2⅓ Jahre,	23⅓ J.
"	$8\frac{4}{7}$	"	4.2 Tage,	42 Tage,	14 Mon.,	140 M.
Monatl.	1½	"	2 Tage,	20 Tage,	200 T.,	2000 T.
"	2	"	1.5 Tage,	15 Tage,	5 Mon.,	50 Mon.
"	2½	"	1.2 Tage,	12 Tage,	4 Mon.,	40 Mon.
"	1	"	3 Tage,	1 Mon.,	10 M.,	100 M.
"	¾	"	4 Tage,	40 Tage,	400 T.,	4000 T.
"	1¼	"	2.4 Tage,	24 Tage,	8 Mon.,	80 Mon.

$ 8	7	6	0.	75	
	3	4	1.	80	
96	9	4	3.	78	
50	5	0	0.	25	
7	0	0	0.	00	u. s. w.

Alle Exempel lassen sich so ausrechnen ohne eine Ziffer zu schreiben. Die Angaben der Zeit werden in einer Minute gefunden durch Anwendung der folgenden Regel, die auf Seite 17—19 dieses Buches vollständig erörtert und für jede Berechnung erklärt ist.

Regel für alle Raten. Drehe die Rate um, hänge eine Null an und versetze den Decimalpunkt.

Dreht man die Rate um, so hat man die Zeit, in welcher ein Dollar einen Cent verdient; den Decimalpunkt in der Geldsumme um zwei Stellen zur Linken gerückt, ergibt die Interessen der betreffenden Summe für diese Zeit und Rate. Das Anfügen einer Null zur Zeit ergibt die Zeit, in welcher ein Dollar zehn Cents verdient, und das Versetzen des Punktes in der Geldsumme um eine Stelle zur Linken ergibt die Interessen der Summe für diese Zeit. Die Zeitangabe durch zehn dividirt durch Setzen des Decimalpunktes, ergibt die Zeit, in welcher ein Dollar eine Mille verdient, und in der Geldsumme den Punkt um drei Stellen zur Linken versetzt, ergibt die Interessen der betreffenden Summe für diese Zeit. Man vergrößere oder verkleinere die Summen der verlangten Zeit anpassend.

Nach derselben Regel lassen sich die Interessen berechnen, wenn man den Decimalpunkt in der Zeit oder der Zahl der Tage versetzt. Das Umdrehen der Rate ergibt die Zeit, in der ein Dollar einen Cent verdient, und das Zufügen einer Null ergibt die Zeit, in welcher ein Dollar zehn Cents verdient u. s. w. Da nun zu der Rate von 8 per Cent jährlich ein Dollar in 45 Tagen einen Cent verdient, so verdienen auch 45 Dollar in einem Tage einen Cent, und der hundertste Theil der Tage ist gleich den Interessen für diese 45 Dollar; 450 Dollar verdienen zehn Cents in einem Tage, und der zehnte Theil der Tage zeigt die Interessen für diese

Summe an; $4.50 verdienen in einem Tage eine Mille, und der tausendste Theil der Tage bildet die Interessen u. s. w. Das Umdrehen der Rate ergibt die Summe der Dollar, welche einen Cent in einem Tage verdient; dieser Zahl eine Null zugesetzt, ergibt die Summe, welche zehn Cents in einem Tage verdient; in derselbe Zahl den Decimalpunkt um eine Stelle zur Linken versetzt, ergibt die Summe, welche eine Mille in einem Tage verdient, und in der Zahl der Tage den Decimalpunkt um drei Stellen zur Linken versetzt, ergibt die Interessen für diese Summe.

Jährlich	8	pr. Ct.	$4.50	$45.00	$450.00	$4500.00
"	9	"	4.00	40.00	400.00	4000.00
"	12	"	3.00	30.00	300.00	3000.00
"	6	"	6.00	60.00	600.00	6000.00
"	3	"	12.00	120.00	1200.00	12000.00
"	7½	"	4.80	48.00	480.00	4800.00
"	4$\frac{2}{7}$	"	8.40	84.00	840.00	8400.00
Monatl.	1$\frac{1}{3}$	"	2.00	20.00	200.00	2000.00
"	1$\frac{1}{4}$	"	2.40	24.00	240.00	2400.00
"	2	"	1.50	15.00	150.00	1500.00
"	3	"	1.00	10.00	100.00	1000.00

Die Interessen findet man nun durch Versetzen des Decimalpunktes in der Zahl der Tage und Vorsetzen des Dollarzeichens.

$1	2	3	6
	9	8	7
	4	3	3
	1	1	1
		5	8
	1	4	4
1	3	8	9
		1	5
		1	0
			1

APPENDIX.

Containing Definitions, and Explanations of Terms, &c.

The symbol =, is called the sign of *equality;* and denotes that the quantities between which it is placed, are equal or equivalent to each other. Thus, \$10=1000 cents, which is read, ten dollars equals ten hundred cents.

The symbol +, is called the sign of *addition*, or plus; and denotes that the numbers between which it is placed are to be added together. Thus, 3+4=7.

The symbol —, is called the sign of *subtraction*, or minus; and denotes that the number which is placed on the right of it is to be subtracted from the number on the left. Thus, 7—3=4, is read, seven minus three equals four.

The symbol ×, is called the sign of *multiplication;* and denotes that the numbers between which it is placed are to be multiplied together. Thus, 3×4=12, is read, three multiplied by four equals twelve.

The symbol ÷, is called the sign of *division;* and denotes that the number on the left of it is to be divided by the number on the right. Thus, 8÷4=2, is read, eight divided by four equals two.

When numbers are enclosed in parenthesis, they are to be treated as a single number.

Thus, $(4+6)\div 2=5$, indicates that the sum of 4 and 6 is to be divided by 2.

The same thing may be expressed by drawing a line over the numbers; thus, $\overline{3+4}\times 2=14$.

Definitions of Terms.

An axiom is a self-evident truth.

A demonstration is a train of logical arguments brought to a conclusion.

A theorem is a truth which becomes evident by means of a demonstration.

A problem is a question proposed, which requires a solution.

Axioms.

1. Things which are equal to the same thing, are equal to each other.
2. If equals be added to equals, the wholes will be equal.
3. If equals be taken from equals, the remainders will be equal.
4. If equals be added to unequals, the wholes will be unequal.
5. If equals be taken from unequals, the remainders will be unequal.
6. Things which are doubles of equal things, are equal to each other.
7. Things which are halves of equal things, are equal to each other.
8. The whole is greater than any of its parts.

9. The whole is equal to the sum of all its parts.

10. All right angles are equal to each other.

11. From one point to another only one straight line can be drawn.

12. A straight line is the shortest distance between two points.

One of the most important and practical of my discoveries, is the new alphabet of numbers, and may be explalned thus: Let ten represent the base of our system of notation; and since ten divided by ten equals the zero power of ten, also ten divided by ten equals one one, we have, according to axiom first, the zero power of the base equivaleut to the first numerical symbol.

Thus, $10^1 \div 10^1 = 10^0$. To divide, subtract the power of the divisor from the power of the dividend.

And, $\frac{10}{10} = \frac{1}{1}.$ 2d. Divide dividend and divisor by ten.

Hence, according to axiom first $10^0 = \frac{1}{1}.$

The second symbol is twice the first; or, $\frac{2}{1}$. The third is three times the first; or, $\frac{3}{1}$, &c., to the last, which is $\frac{0}{1}$ ones. The first character, $\frac{1}{1}$ may represent any finite quantity in the solution of a problem. Thus, A and B own 100 acres of land; B owns twice as much as A, how much does each own?

Solution.—Let $\frac{1}{1}$ represent what A owns, since B owns twice as much as A, $\frac{2}{1}$ will represent what B owns; and $\frac{1}{1}+\frac{2}{1}$, or $\frac{3}{1}=100$ acres, and $\frac{1}{1}$, what A owns, is one third of 100, or $33\frac{1}{3}$ acres; $\frac{2}{1}$, what B owns, is twice $33\frac{1}{3}$, or $66\frac{2}{3}$ acres.

The last symbol has no value when it stands alone, but when inverted becomes $\frac{1}{0}=\infty$, the symbol of an infinite quantity.

The rules of Square and Cube Root are the most important in this book.

The office of square root is to find the linear edge of a square.

The sign of Square Root is indicated by $(\ )^{\frac{1}{2}}$, and denotes, when placed over a number, that the square root of the number is to be extracted. Thus, $(16)^{\frac{1}{2}}=4$, is read, the square root of 16 equals 4.

The office of cube root is to find the linear edge of a cube, and is indicated by $(\ )^{\frac{1}{3}}$, and when placed over a number, denotes that the cube root of the nnmber is to be extracted. Thus, $(8)^{\frac{1}{3}}$, is read, the cube root of 8 equals 2.

The methods of computing interest, marking

goods, measuring land, grain, and extracting square and cube roots, are performed decimally, and need no further explanation.

The value of everything bought and sold, is calculated by the light of the same principle; thus, the value of 1321 lbs. of hay at $12 per ton, is found by removing the decimal point three places toward the left on the number of pounds, and multiplying the result by half the price per ton; thus, $1.321 \times 6 = \$7.926$, and we have the answer in dollars and cents.

Note.—Removing the point three places reduces the number to units of a thousand, and dividing the price of a ton by 2 gives the cost of 1000 lbs., and multiplying the number of thousand lbs. by the price of 1000 lbs. invariably gives the cost. It may also be found by removing the point one place toward the left on the cost, and dividing the result by two, which gives the cost of 100 lbs., and removing the point two places on the number of lbs. to reduce to units of 100, and the product of the number of hundred lbs. by the price of 100 lbs. will give the cost of everything bought and sold by the ton.

The above rules apply universally when the number of pounds in a ton is 2000.

The rule for buying and selling articles when 2240 pounds are considered a ton, is similar to

the foregoing one, because 2000 weight of coal makes a ton, but a hundred weight of coal is 112 pounds. Now, removing the point one place to the left on the price per ton, $12, and dividing by two, gives the cost of one hundred weight. By increasing or diminishing the cost of one hundred weight, the cost of any number of pounds is readily found.

One of the California Big Trees, 400 feet high, was broken in a storm, the butt remaining on the stump, the top resting on the ground 150 feet from the root. What is the length of the part broken off, and what the hight of the stump?

Rule and Solution for this, and all similar Problems.—One-half the length of the tree plus the square of the base divided by the the length of the tree equals the part broken off. One-half the length minus the square of the base divided by the length equals the hight of the stump.

Thus, $\frac{1}{2}\times\left(400+\frac{\overline{150}^2}{400}\right)=228\frac{1}{8}$ feet, the length of the part broken off.

NOTE.—The above tree had bark 22 in. thick. Diameter of tree, 35 feet.

What the product of £19, 19s. 11d. 3 far. multiplied by the same number?

£19, 19s. 11d. 3f.—1 far. the complement.

20

£399, 19s. 2d. $\frac{1}{960}$ far. Simply multiply by 20, the base, and add the square of £$\frac{1}{960}$, the complement.

THE LAW OF POINTS IN DECIMAL FRACTIONS.

The power of the whole number is always positive. The power of the decimal part is always negative. Since the power indicates the place of the decimal point, we can not err in fixing it, because we always point off as many places for the decimal part of the answer as is indicated by the negative power in the product in multiplication; and in division as many as is indicated by the negative power of the quotient. Hence, we have the law add the power of the multiplier to the power of the multiplicand, the sum indicates the number of decimal places in the product of any two numbers.

Subtract the power of the divisor from the power of the dividend, the remainder always indicates the number of decimal places to be pointed off in the quotient. The multiplication and division are performed as in whole numbers.

The above rules become very clear when you examine the proof of the fnndamental rule of arithmetic, explained in fore part of this book. We will give one or two illustrations here: You have seen that the value of a whole number is found by multiplying the number by the zero power of the base of numbers; thus 144 is 144 times the zero power of ten, or one hundred and forty-four ones. Now, one hundred and forty-four thousandths, its value is found by multiplying 144 by the base minus the third power, or the third power of the base

of numbers inverted which gives $\frac{144}{1000}$; because ten minus the first power is one tenth, or simply ten inverted; ten minus the second power, is the square of ten inverted; ten minus the third power, is the third power of ten inverted, or $\frac{1}{1000}$, simply annexiug ciphers to raise the base to the required power.

Multiply 1.03 by 9.7, the product is 9.991.

RULE.—Multiply as in whole numbers, and point off as many places in the product for decimals as is found in the sum of the power of multiplier and multiplicand in all examples. You observe in the above example the power of the multiplier is minus one, the power of the multiplicand is minus two, their sum minus three. Hence, the unity of the decimal in the above example is the third power of ten inverted, or one thousandth; hence point off three places.

If the power of the product is minus one the unity of the decimal part in the answer is one tenth, and we point off one place for decimals; if minus two, the unity is one hundredth, and we point off two places; if minus three, three places; minus four, four places, for decimals, in the answer, &c.

DIVISION OF DECIMAL FRACTIONS.

RULE.—Subtract the power of the divisor from the power of the dividend, the quotient indicates the value of the *unity* in the decimal part of the answer, or the number of places to point off in the quotient for decimals.

NOTE.—Divide as in whole numbers, you change the sign of the power of the divisor and add it to the power of the dividend, the sum indicates the power of the quotient, or the place of the decimal point.

N. B.—When the sum is positive one, you annex one cypher; positive 2, annex two cyphers; positive 3, annex three cyphers to the quotient, &c.

When negative,—1, point off one place for decimals; negative,—2, point off two places for decimals; when negative,—3, point off three places for decimals in the quotient, &c.

Example.—Divide 12 by 1.2, dividing as in whole numbers, we get one for a quotient, but the power of the quotient is positive one; hence, annex a cypher and we have 10 for the answer.

Example 2d.—Divide .001 by 1000. We annex three cyphers to the dividend, making the power of the dividend negative,—6, the power of the divisor is positive 0; subtracting, leaves the power of the quotient negative,—6. Hence, point off six places in the quotient, and we have .000001, the correct quotient.

THE LAW OF DIVISORS IN CUBE ROOT.

The trial divisor is always three sides of the cube on which you are making the additions. Add to the trial divisor one side of each of the corner additions and one side of the small cube for the true divisor. Each trial divisor is found by adding to the true divisor, as it occurs, one side of each of the corner additions and two sides of the small cube, as represented in the engraving, page 47, and explained on page 49 of this book.

To find the cost of any number of feet of lumber, remove the decimal point three places to the

left on the number of feet, and multiply by the cost of one thousand feet in all examples.

NOTE.—Remove the point three places to reduce the number of feet to units of a thousand, and multiply the number of thousand feet by the price of one thousand gives the cost in all examples. Thus : 3121 feet of lumber at $8 a thousand ; removing the point three places, we have 3,121, multiplying by 8, the price of one thousand, we have

$$\begin{array}{r} 3{,}121 \\ 8 \\ \hline \$24.968 \end{array}$$

To multiply ones by ones, thus : 11111111×11111111=123456787654321, simply count to eight and back to one.

To multiply threes by threes, thus : 33333333×33333333=1111111088888889, &c., for any number of threes.

To multiply sixes by sixes, thus : 666666666×666666666=444444443555555556 ; the same order must be observed in any number of sixes.

ANALYSIS

OF

HENDERSON'S LIGHTNING CALCULATOR.

INTRODUCTION.

RIPE FRUIT IS WHOLESOME.

This work falls into your hand like a ripe pear. Years of careful study and close investigation have developed this beautiful system of arithmetic.

The rules of the previous editions of this work, have been considered models of simplicity and clearness, by those foremost in thought and science.

We now add many new, concise and practical rules, and the thorough analysis of the whole work; making it the most useful and interesting work on the science of numbers ever offered to the public.

This work, unlike other arithmetics, needs no key, because we give you with it the com-

plete analysis which is the golden key that unlocks all the mysteries of mathematics; the young student that peruses carefully this book and its analysis, is prepared to make understandingly any business calculation with ease and rapidity; besides finding a strong pure light, that strengthens and quickens the mind in every branch of science and business.

This work is a self-instructor needing no tutor to explain, and ought to find its way into every home in christendom.

A gentleman of culture remarked to me that he would not be without this work in his family for one hundred dollars, for it had created an interest in the study of numbers. The domestic circle ought to be the nursery of thought and intellect, as well as that of virtue.

Take care of the education of the family circle and it will take care of the common school, and the common school the academy, and the academy, the college and university. The fire must be kindled in the lowest halls of learning, then the heat and light ascends to every part of the super-structure. Take care of the fountain, and you have a permanent and pure stream.

The happiness, wealth and lasting pros-

perity of state and nation depend almost entirely on the culture and power of labor of the people, for labor is the only universal currency.

Time is too valuable to be employed in studying wrong and long rules that only confuse the mind, and make the child stupid that might be in the light of the rules of this book, as brilliant as the morning sun.

Traveling in a crooked path never makes it straight. Repeating a lie never makes it the truth.

Practicing wrong rules for hundreds of years, and understanding ever so well every crook and turn in them never makes them one whit more practical to the beginner. Early education is something like that kind of ink which when first put on paper, is scarcely visible, but it becomes blacker and blacker, and now so black you may burn it to cinder and the writing is there legible. Hence we have been careful to present nothing but the right methods of making business calculations, that the young mind may build up its education without any fundamental errors.

We hope this work will be instrumental in eliminating from the minds of the people many false views and wrong rules which are

now in use in the common text-books in our schools.

We are convinced that we have given to the world, and now I speak only of what I am the author, the only rational alphabet of numbers, — the only fundamental proof of addition, — the only right and lightning method of calculating interest, — the only right method of evolution, or the extraction of roots, the law of trial and true divisors, — the law of the decimal point in division and multiplication of decimal fractions, and the most complete and progressive system of *mental* calisthenics, — the only right method of treating fractions, — the right rule of gold and currency, measuring land, lumber, stone, hay, grain, &c., &c.

I am assured that the right study of this work will quicken the perception, strengthen the intellect and brighten the mind, as surely as the rising sun floods the mountain, hill and valley with light and beauty.

J. A. HENDERSON,

Author and Proprietor.

ANALYSIS.

Example 1.—How many tons of iron would it take to build a railroad of 3000 miles, if one yard weighs 35 pounds?

Statement.—$\frac{11}{7} \times 35 \times 3000 = 11 \times 5 \times 3000 =$ 165000 tons, or $55 \times 3000 = 165000$ tons.

Example 2.—How many tons of iron would it take to build a railroad 233 miles?

Solution.—Since it takes 55 tons to build one mile it would take for 233, $233 \times 55 = 12815$ tons.

Example 3.—If there are 80000 miles of railroad in U. S., and one yard of rail weighs 35 pounds, how many tons did it take? Simply $55 \times 80000 =$ 4400000 tons.

Every example is stated in the same form, it matters not what one yard weighs. If one yard weighs 40 pounds, the statement is $\frac{11}{7} \times 40 \times$ by the number of miles, and you have the number of tons.

Example 4.—If there are 1400000 miles of railway on this globe at present, and one yard of rail weighs 40 pounds, how many tons of iron would it take? Simply $\frac{11}{7} \times 40 \times 1400000 = 11 \times 40 \times$ $200000 = 88000000$ tons.

Ratio is that which is expressed by the quotient of one number divided by another; and the most convenient method of expressing it, is in the form of a fraction.

The numbers 1760 and 2240. The ratio is $\frac{1760}{2240}$ $=\frac{11}{14}$, $\frac{11}{14}$ is the direct ratio of the two numbers, and $\frac{14}{11}$ the inverted or reciprocal ratio of the numbers. From the above illustration a rule is formed by which the number of tons of iron it would take to construct any number of miles of railroad is found, for 1760 is the number of yards in one mile, and 2240 is the number of pounds in one ton of iron, and the ratio is $\frac{11}{14}$, hence: if one yard of rail weighs 35 pounds, $\frac{11}{14} \times 35 =$ the number of tons that it would take to construct one mile of the railroad (one side of the track), because the unity of the ratio is a ton, and if a yard weighed only one pound, it would take $\frac{11}{14}$ of a ton to build a mile, but since a yard weighs 35 pounds it would take 35 times $\frac{11}{14}$ of a ton, and since the track has two sides, we multiply $\frac{11}{14}$ by 2, and have $\frac{11}{7}$, the number of tons that it would take to build the double track one mile, providing a yard weighed but a pound. Hence the rule for all examples, *multiply the expression $\frac{11}{7}$ by the number of pounds one yard weighs and the product is the number of tons it takes for one mile, multiply the number of miles by what it takes for one mile and you have in a moment the number of tons it takes for any number of miles.*

Unity is the basis of every whole number, and is represented in the denominator of every fraction divided into equal parts.

Its office is different from that of the unit because it is detached from the unit or number as indicated

in my improved alphabet of numbers, and aids in simplifying and abbreviating nearly all of the operation in the science of the mathematics, $\frac{1}{1}$ $\frac{2}{1}$ $\frac{3}{1}$ $\frac{4}{1}$ $\frac{5}{1}$ $\frac{6}{1}$ $\frac{7}{1}$ $\frac{8}{1}$ $\frac{9}{1}$ and $\frac{0}{1}$, one fundamental error that mystifies the operations, rules, and statement of problems is omiting the unity or base of the number which is the primary starting point in every investigation. Instead of calling the usual characters representing numbers digits.

The digit in numbers is the shoot or the base of the number and we have ten because there are $\frac{10}{1}$ in the base of our system of notation, and the one below the line is the unity or digit.

Example 1.—James and John have $96, John has 7 times as many as James, how many dollars has each.

Solution.—Let one one represent what James has, then seven ones represent what John has, or eight ones equal ninety six dollars, and one one equals twelve, and seven ones equal eighty four dollars, or $\frac{8}{1} = \$96$, hence $\frac{1}{1} = \$12$ and $\frac{7}{1} = \$84$.

Example 2.—A and B are worth $1000, B is worth 4 times as much as A, what is each worth?

Solution.—$\frac{1}{1}$ represents what A is worth, $\frac{4}{1}$ what B is worth, hence $\frac{5}{1}$ or 5 times what A is worth is $1000, and $\frac{1}{1}$ what A is worth is one fifth of $1000 or $200, and B is worth $\frac{4}{1}$ or $800.

Example 3.—A B and C are worth $1200, B is worth twice as much as A, and C is worth as much as A and B, what is each one worth?

Solution.—Let $\frac{1}{1}$ represent what A is worth, then B is worth $\frac{2}{1}$ and C is worth $\frac{3}{1}$, hence $\frac{6}{1}$ = \$1200, and $\frac{1}{1}$ = \$200 and $\frac{2}{1}$ = \$400 and $\frac{3}{1}$ = \$600.

Example 4.—A B C and D are worth \$10000, B is worth three times what A is worth, and C is worth one half as much as A and B, and D is worth two thirds as much as A B and C, what is each worth?

Solution. Let $\frac{1}{1}$ = what A is worth then B is worth $\frac{3}{1}$ and C is worth $\frac{2}{1}$ and D is worth $\frac{4}{1}$, hence $\frac{10}{1}$ or 10 times what A is worth equals \$10000, and $\frac{1}{1}$ = \$1000 B is worth $\frac{3}{1}$ or \$3000 and C $\frac{2}{1}$ or \$2000 and D $\frac{4}{1}$ or \$4000.

Example 5.—If a field contains 4800 rods, its length is three times its width, what is its length and width?

Solution.—Let $\frac{1}{1}$ represent the width, then three times $(\frac{1}{1})^2 = 4800$, and $(\frac{1}{1})^2 = 1600$, and $\frac{1}{1} = 40$ the width of the field, the length is $\frac{3}{1}$ or 120 rods.

Example 6.—A field contains 1600 rods, its length is four times its width, what is its length and width?

Solution.—Let $\frac{1}{1}$ represent its width, then four times $\frac{1}{1}$ square equals 1600, and $\frac{1}{1}$ square equals 400, and $\frac{1}{1}$ equals 20 rods the width of the field, the length is four times twenty or 80 rods.

The method of performing the indicated oper-ation in the statement of an example is of some importance, hence we give a hint or two before presenting the analysis of fractions.

Example 7.—What is the quotient of one hundred and forty four multiplied by seventy-nine and divided by seventy-two?

Statement. $\frac{144\times79}{72}$

Solution.—Dividing the first term of the dividend 144 by 72 the divisor we get 2, and $2\times79=158$, quotient or answer.

Example 8.—What is the quotient of 1728×63 and divided by 144?

Statement. $\frac{1728\times63}{144}$

Rejecting factors common to dividend and divisor we have 12×63 or 756.

Example 9.—What is the result of 128×34 and divided by 32×8?

Statement. $\frac{128\times34}{32\times8}$

Solution.—Dividing the terms of the dividend and the divisor first by 32 then by 4 and we have $\frac{34}{2}$ or 17.

Example 10.—What is the product of $4\times8\times4\times49$, and divided by 128?

Statement. $\frac{4\times8\times4\times49}{128}$

Dividing dividend and divisor by 128 we have 49.

Example 11.—What is the quotient of 15×17, divided by $7\frac{1}{2}$?

Statement. $\frac{15\times17}{7\frac{1}{2}}$

Operation, dividing dividend and divisor by $7\frac{1}{2}$ and we have 2×17 or 34.

Example 12.—What is the result of $\frac{15\times31\times47}{15\frac{1}{2}\times23\frac{1}{2}}$

Solution. Dividing both dividend and divisor by the terms of the divisor we have $15\times2\times2$ or 60.

Example 13.—What is the result of $\frac{17\frac{1}{2}\times36\frac{1}{3}\times24}{35\times18\frac{1}{6}}$

Solution. Dividing dividend and divisor by $17\frac{1}{2}$ then by $18\frac{1}{6}$ and we have $\frac{1\times2\times24}{2\times1}$ or 24.

ANALYSIS OF FRACTIONS.

There are two terms that must be examined in the analysis of fractions which are called the numerator and denominator, the numerator represents how many parts of the unity are taken, and is written above the line. The denominator shows into how many parts the unity is divided, and is written below the line, thus $\frac{3}{4}$ of a dollar, the 4 the denominator of the fraction, indicates into how many parts the dollar is divided, and the 3, the numerator, represents how many parts are taken. $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{9}$ $\frac{1}{10}$ $\frac{1}{11}$ $\frac{1}{12}$ &c., are read one-one, one-half, one-third, one-fourth, one-fifth, &c.

Now the value of the denominator of any fraction is always equivalent to unity because dividing it into many or few parts does not increase or diminish its value, hence the value of the denominator of any fraction is fixed and changeable only in form, its value being always that of the unity under consideration. The numerator of a fraction

may be equivalent to a unit or greater or less than a unit, according to what is expressed by the fraction, thus $\frac{4}{4}$ of a yard of cloth are equivalent to one yard, $\frac{7}{8}$ less than a yard, while $\frac{9}{8}$ are more than a yard.

The expression $\frac{4}{4}$ the number of parts that unity is divided into, is expressed in the numerator, hence its value is one, the expression $\frac{7}{8}$ the numerator is less than the denominator, hence its value is less than one, the expression $\frac{9}{8}$ the numerator is greater than the denominator, hence its value is greater than one, hence a fraction may represent in value what is equivalent to one, more or less than one, therefore a fraction is a collection of one or more of the equal parts of unity. The base of every whole number is unity as we have represented in the alphabet of numbers, and the primary base of every fraction is unity, hence fractions are handled by the same rules that we have been using to perform operations in whole numbers. Thus to divide any whole number by any other whole number we divide by multiplying the base of the whole number by the given divisor or dividing the given number by the divisor.

ANALYSIS OF ADDITION OF FRACTIONS.

Example 1.—What is the sum of one-half and one-third?

Statement.—$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, answer.

Thus the sum of two and three is the numerator

of the answer, and the product of two and three the demoninator of the answer.

Example 2.—What is the sum of one-third and one-fourth?

Statement.—$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$.

The sum of three and four is seven the numerator of the answer, and the product of three and four is twelve the denominator of the answer.

Example 3.—What is the sum of one-fifth and one-sixth?

Statement.—$\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$.

Place the sum of the denominators over the product of the denominators.

Example 4.—What is the sum of one-seventh and one-eight?

Statement.—$\frac{1}{7} + \frac{1}{8} = \frac{15}{56}$.

The sum of seven and eight is the numerator of the answer, and their product the denominator of the answer.

Example 5.—What is the sum of twelve and one-seventh and nine and one-eight?

Statement.—$12\frac{1}{7} + 9\frac{1}{8} = 21\frac{15}{56}$.

Simply the sum of the whole numbers is twenty one, and the sum of the fractions fifteen-fiftysixths.

Example 6.—What is the sum of four and one-half and seven and one-third?

Statement.—$4\frac{1}{2} + 7\frac{1}{3} = 11\frac{5}{6}$.

The sum of the whole numbers is eleven and the sum of the fractions is five-sixths.

Example 7.—What is the sum of $13\frac{1}{9}+5\frac{1}{10}=18\frac{19}{90}$.

The sum of the whole numbers is 18, and the sum of the fraction is $\frac{19}{90}$.

RULE 1.—Reduce the fractions to a common base, add their numerators and place the sum over the common base.

Example 8.—What is the sum of $\frac{1}{2}$ and $\frac{2}{3}$?

Solution.—The common base is sixths, $\frac{1}{2}=\frac{3}{6}$ and $\frac{2}{3}=\frac{4}{6}$, their sum is $\frac{7}{6}=1\frac{1}{6}$ answer.

Example 9.—What is the sum of $\frac{3}{4}$ and $\frac{7}{8}$?

The common base is eighths, $\frac{3}{4}=\frac{6}{8}$, and $\frac{6}{8}+\frac{7}{8}=\frac{13}{8}=1\frac{5}{8}$ answer.

Example 10.—What is the sum of $\frac{3}{4}+\frac{2}{3}+\frac{5}{6}$?

The common base of these fractions is 12, hence we have $\frac{9+8+10}{12}=\frac{27}{12}=2\frac{1}{4}$, answer.

Example 11.—What is the sum of $\frac{1}{8}+\frac{2}{3}+\frac{5}{12}$?

The common base of these fractions is 24, hence we have $\frac{3+16+10}{24}=\frac{29}{24}=1\frac{5}{24}$, answer.

Example 12.—What is the sum of $\frac{3}{4}+\frac{5}{8}+\frac{2}{3}$?

The common base is 24, hence we have $\frac{3}{4}+\frac{5}{8}+\frac{2}{3}=\frac{18+15+16}{24}=2\frac{1}{24}$, answer.

Example 13.—What is the sum of $\frac{7}{12}+\frac{1}{8}+\frac{2}{3}$?

The common base is 24, hence we have $\frac{7}{12}+\frac{1}{8}+\frac{2}{3}=\frac{14+3+16}{24}=1\frac{3}{8}$, answer.

Example 14.—What is the sum of $\frac{17}{48} + \frac{1}{12} + \frac{5}{8}$?

The common base of these fractions is 48, hence the sum is $\dfrac{17+4+30}{48} = 1\frac{1}{16}$, answer.

Example 15.—What is the sum of $\frac{39}{100}+\frac{1}{5}+\frac{1}{4}+\frac{1}{25}$?

The common base of these fractions is 100, hence the sum is $\dfrac{39+20+25+4}{100} = \frac{88}{100}$, answer.

Rule 2.—Plan of reducing fractions to a common base.

Rule, the product of each numerator into all the denominators except its own for numerators, and all the denominators together for the common base of the sum of the fractions.

Example 16.—What is the sum of $\frac{2}{9} + \frac{1}{2} + \frac{1}{4}$?

Solution.—
$$\begin{array}{r}
2 \times 2 \times 4 = 16 \\
1 \times 9 \times 4 = 36 \\
1 \times 9 \times 2 = 18 \\
\hline
70 \\
9 \times 2 \times 4 = \overline{72}
\end{array} = \frac{35}{36}, \text{ answer}$$

Example 17.—What is the sum of $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$?

Solution.—
$$\begin{array}{r}
1 \times 3 \times 4 = 12 \\
1 \times 2 \times 4 = 8 \\
1 \times 2 \times 3 = 6 \\
\hline
26 \\
2 \times 3 \times 4 = \overline{24}
\end{array} = 1\frac{1}{12}, \text{ answer.}$$

Example 18.—What is the sum of $\frac{3}{4} + \frac{2}{3} + \frac{1}{2}$?

Solution.—
$$\begin{array}{r}
3 \times 3 \times 2 = 18 \\
2 \times 4 \times 2 = 16 \\
1 \times 4 \times 3 = 12 \\
\hline
46 \\
4 \times 3 \times 2 = \overline{24}
\end{array} = 1\frac{11}{12} \text{ answer.}$$

Example 19.—What is the sum of $\frac{5}{8}+\frac{3}{4}+\frac{1}{2}$?

Solution.—$5\times4\times2=40$

$$3\times8\times2=48$$
$$1\times8\times4=32$$
$$\frac{120}{8\times4\times2=64}=1\tfrac{7}{8},\text{ answer.}$$

RULE 3.—Multiply each fraction by the least common base of all the fractions to be added, write their sum over the least common base.

Example 20.—What is the sum of $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$?

The least common base of these fractions is 12.

Solution.—$12\times\frac{2}{3}=8$

$$12\times\frac{3}{4}=9$$
$$12\times\frac{5}{6}=10$$
$$\frac{27}{12}=2\tfrac{1}{4},\text{ answer.}$$

Example 21.—What is the sum of $\frac{18}{24}+\frac{5}{8}+\frac{10}{12}$?

The least common base is 24.

Solution.—$\frac{18}{24}\times24=24$

$$\frac{5}{8}\times24=15$$
$$\frac{10}{12}\times24=20$$
$$\frac{59}{24}=2\tfrac{11}{24},\text{ answer.}$$

When the least common base of the fractions cannot be found by observation, place the bases of the fractions on the same line and divide by the least number that will divide two or more of them; setting down the quotient and the undivided bases on the line below; then divide as before, until there is no number greater than one that will

divide any two of the numbers. Multiply the divisors and the numbers on the lower line together and their product will be the least common base of the fractions.

ANALYSIS OF SUBTRACTION OF FRACTIONS.

Fractions must be reduced to the same fractional unit first, then subtract the subtrahend from the minuend, or reduce the fractions to a common base, and subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference over the common base.

Example 1.—From $\frac{1}{3}$ take $\frac{1}{4}$?

The common base is 12, $\frac{1}{3}$ is $\frac{4}{12}$, $\frac{1}{4}$ is $\frac{3}{12}$, $\frac{4}{12} - \frac{3}{12} = \frac{1}{12}$, answer. Or $4 - 3$ over 4×3 is $\frac{1}{12}$.

Example 2.—From $\frac{1}{4}$ take $\frac{1}{5}$?

$\frac{1}{4}$ is $\frac{5}{20}$, and $\frac{1}{5}$ is $\frac{4}{20}$, $\frac{4}{20} - \frac{5}{20}$ is $\frac{1}{20}$, answer.

Example 3.—From $\frac{1}{5}$ take $\frac{1}{6}$?

The common base here is 30, $\frac{1}{5}$ is $\frac{6}{30}$, and $\frac{1}{6}$ is $\frac{5}{30}$, $\frac{6}{30} - \frac{5}{30} = \frac{1}{30}$, answer. Or simply $6 - 5$ over $5 \times 6 = \frac{1}{30}$, answer.

Example 4.—From $\frac{1}{7}$ take $\frac{1}{8}$?

Solution.—$\frac{1}{7}$ is $\frac{8}{56}$, and $\frac{1}{8}$ is $\frac{7}{56}$, $\frac{8-7}{56} = \frac{1}{56}$, ans.

Example 5.—From $\frac{1}{8}$ take $\frac{1}{9}$?

Solution.—$\frac{9-8}{8 \times 9} = \frac{1}{72}$, answer.

Example 6.—From $\frac{1}{10}$ take $\frac{1}{11}$?

Solution.—11 — 10, over 10 times 11, or $\frac{1}{110}$, ans.

Example 7.—From $\frac{1}{12}$ take $\frac{1}{14}$?

Solution.—14 — 12 over 12 times 14, or $\frac{2}{168} = \frac{1}{84}$, answer.

RULE OF SUBTRACTION OF FRACTIONS.

Rule.—Reduce the fractions to a common base, or to the least common base, and place the difference of their numerators over the common base.

Example 8.—From $\frac{3}{4}$ take $\frac{4}{10}$?

Solution.—The least common base is 20, $\frac{3}{4} = \frac{15}{20}$, and $\frac{4}{10} = \frac{8}{20}$, $\frac{15 - 8}{20} = \frac{7}{20}$, answer.

Example 9.—From $\frac{5}{7}$ take $\frac{8}{14}$?

The least common base is 14; $\frac{5}{7} = \frac{10}{14}$, and $\frac{10}{14} - \frac{8}{14}, = \frac{2}{14}$, or $\frac{1}{7}$, answer.

Example 10.—From $\frac{3}{4}$ take $\frac{5}{8}$?

The least common base is 8; $\frac{3}{4} = \frac{6}{8}, \frac{6 - 5}{8} = \frac{1}{8}$.

Example 11.—From $8\frac{1}{3}$ take $5\frac{1}{5}$?

Write whole numbers under whole numbers, and fractions under fractions, subtract fractions from fractions, and whole numbers from whole numbers.

$$\begin{array}{r} 8\frac{1}{3} \\ 5\frac{1}{5} \\ \hline 3\frac{2}{15} \end{array}$$

The least common base is 15; $\frac{5 - 3}{15} = \frac{2}{15}$, and $8 - 5 = 3$.

$3\frac{2}{15}$, answer.

Example 12.—From $237\frac{5}{9}$ take $78\frac{6}{9}$?

$$\begin{array}{r} 237\frac{5}{9} \\ 78\frac{6}{9} \\ \hline 158\frac{8}{9} \end{array} \text{ answer.}$$

Example 13.—From $128\frac{7}{8}$ take $33\frac{3}{4}$?

$$\begin{array}{r} 128\frac{7}{8} \\ 33\frac{3}{4} \\ \hline 95\frac{1}{8} \end{array} \text{ answer.}$$

ANALYSIS OF MULTIPLICATION OF FRACTIONS.

RULE.—Multiply together the numerators for the numerator of the answer, and the denominators together for the denominator of the answer, observing to reject factors common to the numerator and denominator.

Example 1.—What is the product of $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$?

$\frac{2}{3} \times \frac{3}{4} \times \frac{5}{6} = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$, answer. Simply rejecting common factors and multiplying the remaining terms in the numerator for the numerator of the answer, and the remaining terms in the denominator for the donominator of the answer.

Example 2.—What is the product of $\frac{3}{4}$, $\frac{4}{3}$ and $\frac{7}{8}$?

Omiting common factors, and we have $\frac{1}{1} \times \frac{1}{1} \times \frac{7}{8} = \frac{7}{8}$, answer.

Example 3. —What is the product of $\frac{2}{3} \times \frac{3}{2} \times \frac{4}{5} \times \frac{5}{4}$?

Omiting common factors, and we have $\frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} = 1$, answer.

Example 4.—What is the product of $\frac{15}{27} \times \frac{27}{19} \times \frac{1}{4}$?

Rejecting common factors, and we have $\frac{15}{1} \times \frac{1}{19} \times \frac{1}{4} = \frac{15}{76}$, answer.

Example 5.—What is the product of $25 \times \frac{87}{57} \times \frac{18}{24} \times \frac{57}{87} \times \frac{24}{18}$?

Rejecting factors common to numerator and denominator, and we have $25 \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} =$ 25, answer.

When the sum of the fractional parts is equal to one, and the whole numbers are equal, add one to the whole number and multiply by the whole number, and annex the product of the fractional parts.

Example 1.—What is the product of $6\frac{7}{8}$ and $6\frac{1}{8}$?

Solution.—$6\frac{7}{8} \times 6\frac{1}{8} = 42\frac{7}{64}$, answer.

Increasing six by one, for the sum of the fractional parts is one, and multiplying by six, we have forty-two, the whole number of the product, and $\frac{7}{8} \times \frac{1}{8} = \frac{7}{64}$ annexed, gives $42\frac{7}{64}$.

Example 2.—$7\frac{3}{5} \times 7\frac{2}{5} = 56\frac{6}{25}$, the product, add one to 7 and we have 8, and $7 \times 8 = 56$, and $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$, hence the product of $7\frac{3}{5}$ and $7\frac{2}{5}$ is $56\frac{6}{25}$.

Example 3.—$5\frac{4}{7} \times 5\frac{3}{7} = 30\frac{12}{49}$, answer. 5 plus 1 multiplied by 5, gives 30, and $\frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$, hence the product of the two numbers is $30\frac{12}{49}$.

This method of multiplying fractions is thoroughly illustrated in my system of mental calisthenics, or mental exercises, commencing at page sixty-two of this work. It is of great worth in the class room; I used it to great advantage in a class of eighty-

nine, while teaching in Delhi Academy, New York, in 1861. Also the following

Rule: The product of any two numbers, whole or fractional, is the square of their mean diminished by the square of half their difference.

Example 1.—What is the product of $4\frac{3}{4}$ and $5\frac{1}{4}$?

Solution.—The mean of these two numbers is five, the square of five is 25, half of the difference of the two numbers is one quarter, and the square of one quarter is $\frac{1}{16}$, hence 25 minus $\frac{1}{16}$ is $24\frac{15}{16}$, the product of $4\frac{3}{4}$ and $5\frac{1}{4}$.

EXAMPLES ILLUSTRATING THE RULE.

What is the product of

$2\frac{1}{2} \times 3\frac{1}{2} = 8\frac{3}{4}$, answer.
$2\frac{1}{3} \times 3\frac{2}{3} = 8\frac{5}{9}$, answer.
$2\frac{5}{6} \times 3\frac{1}{6} = 8\frac{35}{36}$, answer.
$2\frac{8}{9} \times 3\frac{1}{9} = 8\frac{80}{81}$, answer.
$2\frac{6}{7} \times 3\frac{1}{7} = 8\frac{48}{49}$, answer.
$1\frac{1}{2} \times 2\frac{1}{2} = 3\frac{3}{4}$, answer.
$1\frac{2}{3} \times 2\frac{1}{3} = 3\frac{8}{9}$, answer.
$1\frac{7}{8} \times 2\frac{1}{8} = 3\frac{63}{64}$, answer.
$1\frac{5}{6} \times 2\frac{1}{6} = 3\frac{35}{36}$, answer.
$3\frac{1}{2} \times 4\frac{1}{2} = 15\frac{3}{4}$, answer.
$3\frac{3}{4} \times 4\frac{1}{4} = 15\frac{15}{16}$, answer.
$3\frac{5}{6} \times 4\frac{1}{6} = 15\frac{35}{36}$, answer.
$3\frac{7}{8} \times 4\frac{1}{8} = 15\frac{63}{64}$, answer.
$4\frac{1}{2} \times 5\frac{1}{2} = 24\frac{3}{4}$, answer.
$4\frac{2}{3} \times 5\frac{1}{3} = 24\frac{8}{9}$, answer.
$4\frac{6}{7} \times 5\frac{1}{7} = 24\frac{48}{49}$, answer.

$4\frac{1}{8} \times 5\frac{7}{8} = 24\frac{15}{64}$, answer.
$5\frac{1}{2} \times 6\frac{1}{2} = 35\frac{3}{4}$, answer.
$5\frac{2}{3} \times 6\frac{1}{3} = 35\frac{8}{9}$, answer.
$5\frac{5}{6} \times 6\frac{1}{6} = 35\frac{5}{36}$, answer.
$6\frac{1}{2} \times 7\frac{1}{2} = 48\frac{3}{4}$, answer.
$6\frac{8}{9} \times 7\frac{1}{9} = 48\frac{80}{81}$, answer.
$6\frac{6}{7} \times 7\frac{1}{7} = 48\frac{48}{49}$, answer.
$6\frac{1}{7} \times 7\frac{6}{7} = 48\frac{13}{49}$, answer.
$7\frac{1}{2} \times 8\frac{1}{2} = 63\frac{3}{4}$, answer.
$7\frac{2}{3} \times 8\frac{1}{3} = 63\frac{8}{9}$, answer.
$7\frac{8}{9} \times 8\frac{1}{9} = 63\frac{80}{81}$, answer.
$7\frac{1}{9} \times 8\frac{8}{9} = 63\frac{17}{81}$, answer.
&c., &c., &c.

The complement of a number is the difference of that number and some definite number above taken as a base, or multiplier. The supplement of a number is the difference of that number and any definite number below is taken as a base, or multiplier. Hence the rule that I have given of multiplying both, whole and fractional numbers, or squaring whole and fractional numbers. Here I repeat it.

RULE.—Increase the number by its supplement, and multiply by the base and add the square of the supplement; diminish the number by its complement and multiply by the base, and add the square of the complement.

Example 1.—What is the square of $2\frac{1}{2}$?

Solution.—The supplement of $2\frac{1}{2}$ is $\frac{1}{2}$, $2\frac{1}{2}$ plus $\frac{1}{2}$ is 3, and $3 \times 2 = 6$, and 6 plus the square of $\frac{1}{2}$ is $6\frac{1}{4}$, or $2\frac{1}{2} \times 2\frac{1}{2} = 2 \times 3 + \frac{1}{2} \times \frac{1}{2} = 6\frac{1}{4}$.

Example 2.—What is the square of $3\frac{1}{2}$?

$$3\tfrac{1}{2} \times 3\tfrac{1}{2} = 12\tfrac{1}{4}.$$

Example 3.—What is the square of $4\frac{1}{2}$?

$$4\tfrac{1}{2} \times 4\tfrac{1}{2} = 20\tfrac{1}{4}.$$

Example 4.—What is the square of $5\frac{1}{2}$?

$$5\tfrac{1}{2} \times 5\tfrac{1}{2} = 30\tfrac{1}{4}.$$

Example 5.—What is the square of $6\frac{1}{2}$?

$$6\tfrac{1}{2} \times 6\tfrac{1}{2} = 42\tfrac{1}{4}.$$

Example 6.—What is the square of $7\frac{1}{2}$?

$$7\tfrac{1}{2} \times 7\tfrac{1}{2} = 56\tfrac{1}{4}.$$

Example 7.—What is the square of $8\frac{1}{2}$?

$$8\tfrac{1}{2} \times 8\tfrac{1}{2} = 72\tfrac{1}{4}.$$

Example 8.—What is the square of $9\frac{1}{2}$?

$$9\tfrac{1}{2} \times 9\tfrac{1}{2} = 90\tfrac{1}{4}.$$

Example 9.—What is the square of $9\frac{3}{4}$?

Solution.—The complement of $9\frac{3}{4}$ is $\frac{1}{4}$, and $9\frac{3}{4} - \frac{1}{4} = 9\frac{1}{2}$, and ten times $9\frac{1}{2}$ is 95, and the 95 plus the square of $\frac{1}{4}$, is $95\frac{1}{16}$. The above rule applies and is practical in squaring all numbers, whole and fractional.

Problem.—How many feet in a floor eleven feet, eleven and three quarter inches square?

Solution.—The complement of the linear edge of the floor is quarter of an inch, hence by the rule we have (11 ft. $11\frac{3}{4}$ in. $- \frac{1}{4}) \times 12 + (\frac{1}{4})^2 =$ 143 ft. $6\frac{1}{16}$ inches.

ANALYSIS OF DIVISION OF FRACTIONS.

Since the base of every number, whole or fractional, is one in value, inverting any number demonstrates how many times the number is contained in one; for the unity takes the place of the number, and the number the place of the unity.

Example 1.—Divide 5 by 7.

We find by inverting $\frac{7}{1}$, $\frac{1}{7}$, now if 7 is contained in one $\frac{1}{7}$ of a time, it is contained in 5, five times $\frac{1}{7}$, or $\frac{5}{7}$, answer.

Example 2.—Divide 7 by $\frac{3}{4}$.

$\frac{3}{4}$ is contained in one $\frac{4}{3}$ of a time, hence it is contained in $\frac{7}{1}$, $7 \times \frac{4}{3} = \frac{28}{3} = 9\frac{1}{3}$, answer.

Example 3.—Divide 12 by $\frac{1}{2}$.

Inverting $\frac{1}{2}$ we have $\frac{2}{1}$, the number of times $\frac{1}{2}$ is contained in one, hence $\frac{1}{2}$ is contained in 12, 12 times 2, or 24.

Example 4.—Divide $7\frac{1}{2}$ by $\frac{3}{4}$.

Inverting $\frac{3}{4}$ we have $\frac{4}{3}$, the number of times it is contained in one, hence it is contained in $7\frac{1}{2}$ ones $7\frac{1}{2}$ times $\frac{4}{3}$, or $\frac{30}{3} = 10$, answer.

Example 5.—Divide $4\frac{1}{2}$ by 2.

Two is contained in one, one-half of a time, hence it is contained in four and one-half ones, four and one-half times one-half, four times one-half is two and one-half of a half is one-quarter, or thus: $\frac{9}{2} \div 2 = \frac{9}{2} \times \frac{1}{2} = \frac{9}{4} = 2\frac{1}{4}$, answer.

RULE.—Invert the divisor to find how many times it is contained in one, and multiply by the number of ones in the dividend.

Example 6.—Divide 4 by $\frac{5}{8}$.

Solution.—$\frac{5}{8}$ is contained in one $\frac{8}{5}$ of a time, hence it is contained in four-ones, four times eight-fifths, or $\frac{32}{5} = 6\frac{2}{5}$. Inverting any divisor tells how many times it is contained in one, hence multiplying by the number of ones in the dividend gives the correct quotient in all examples in division of fractions.

Example 7.—Divide $\frac{1}{7}$ by $\frac{1}{6}$.

Solution.—$\frac{1}{6}$ is contained in one 6 times, hence it is contained in $\frac{1}{7}$, $\frac{1}{7}$ of 6 ones, or $\frac{6}{7}$ of a time.

Example 8.—Divide 21 by $\frac{3}{4}$.

Solution.—$\frac{3}{4}$ is contained in one $\frac{4}{3}$ of a time, hence it is contained in 21, $\frac{4}{3}$ of 21 or 28, or write the dividend $\frac{21}{1}$, and the divisor $\frac{3}{4}$ directly under it, thus, $\dfrac{\frac{21}{1}}{\frac{3}{4}}$ which is equivalent to inverting the divisor. This brings the factors of the denominator of the answer always between the lines, and the factors of the numerator of the answer always above and below the lines, rejecting factors commen to dividend and divisor. The three is contained in 21 seven times, and $4 \times 7 = 28$.

Example 9.—Divide $\frac{3}{4} \times \frac{5}{7}$ by $\frac{5}{4} \times \frac{3}{8}$.

Solution.—$\dfrac{\frac{3}{4} \times \frac{5}{7}}{\frac{5}{4} \times \frac{3}{8}} = \frac{8}{7}$, simply reject common factors in dividend and divisor and we have $\frac{8}{7}$ for the quotient.

Example 10.—Divide $\frac{5}{12} \times \frac{7}{8} \times \frac{9}{24}$ by $\frac{9}{7} \times \frac{8}{5} \times \frac{5}{24}$.

Statement.—$\dfrac{\frac{5}{12} \times \frac{7}{8} \times \frac{\cancel{9}}{\cancel{24}}}{\frac{\cancel{9}}{7} \times \frac{8}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{24}}} = \dfrac{5 \times 7 \times 7}{12 \times 8 \times 8} = \dfrac{245}{768}$ A.

Simply rejecting the factors common to divisor and dividend, and multiplying the remaining terms between the lines together for the denominator of the answer, and the remaining terms above and below the lines together for the numerator of the answer.

Example 11.—Divide $\frac{27}{29} \times \frac{17}{19}$ by $\frac{27}{19}$.

Statement.—$\dfrac{\frac{\cancel{27}}{29} \times \frac{17}{\cancel{19}}}{\frac{\cancel{27}}{\cancel{19}}} = \dfrac{17}{29}$, answer.

Rejecting the factors 27 and 19 in between the lines, and 27 and 19 outside of the lines and we have $\frac{17}{29}$.

Example 12.—Divide $7\frac{1}{2}$ by $8\frac{1}{4}$.

Statement.—$\dfrac{7\frac{1}{2}}{8\frac{1}{4}} = \frac{30}{33} = \frac{10}{11}$, answer.

Simply multiply divisor and dividend by 4, the least common base of 4 and 2, the denominators of the fractional parts.

Example 13.—Divide $9\frac{5}{12}$ by $3\frac{5}{6}$.

Statement.—$\dfrac{9\frac{5}{12}}{3\frac{5}{6}} = \frac{113}{46} = 2\frac{21}{46}$.

Simply multiply numerator and denominator by 12 the least common base of the denominators of the fractional parts, and reduce to lowest terms.

ANALYSIS OF THE RELATIVE VALUE OF CURRENCY TO GOLD.

RULE.—Take 100 for the numerator and the value of gold for the denominator, and the expression represents the value of one dollar of currency in gold, hence multiply by the number of dollars in currency.

Example 1.—What is the value of fifty dollars of currency in gold, when gold is \$1.25?

Statement.—$\frac{100}{125} \times \frac{50}{1} =$ \$40 in gold.

Solution.—Since there are 100 cents in a dollar, and gold is \$1.25, one dollar in currency is worth $\frac{100}{125}$, or $\frac{4}{5}$ of a dollar in gold, hence 50 dollars in currency is equivalent to $\frac{4}{5}$ of 50, or 40 dollars in gold, when gold is 25 cents above par.

Example 2.—What is the value of 600 dollars of currency in gold, when gold is \$1.12½?

Statement.—$\frac{100}{112\frac{1}{2}} \times \frac{600}{1} = \frac{8}{9} \times \frac{600}{1} = \frac{4800}{9} =$ \$533.33⅓ in gold.

Solution.—Since one dollar in gold is worth 112½ cents in currency one dollar in currency is worth $\frac{100}{112\frac{1}{2}} = \frac{200}{225} = \frac{8}{9}$ of a dollar in gold, hence 600 dollars in currency is worth $\frac{8}{9}$ of 600 or \$533.33⅓ in gold.

Example 3.—What is the value of 99 dollars of currency in gold, when gold is 111⅑?

Statement.—$\frac{100}{111\frac{1}{9}} \times \frac{99}{1} = \frac{9}{10} \times 99 =$ \$89.10 in gold.

Solution.—Since one dollar in gold is worth 111⅑ cents in currency, one dollar in currency is worth $\frac{100}{111\frac{1}{9}}$ or $\frac{9}{10}$ of a dollar in gold, hence 99 dollars in currency is worth $\frac{9}{10} \times 99$ or \$89.10 in gold.

ANALYSIS OF THE RELATIVE VALUE OF GOLD TO CURRENCY.

Example 1.—What is the value of 300 dollars of gold in currency, when currency is 75 cents, or 25 cents below par?

Statement.—$\frac{100}{75} \times \frac{300}{1} = \frac{4}{3} \times \frac{300}{1} =$ \$400.00 in currency.

Solution.—Since one dollar of gold is worth $\frac{100}{75}$, or $\frac{4}{3}$ of a dollar of currency, 300 dollars of gold are worth $\frac{4}{3}$ of 300, or 400 dollars in currency.

Example 2.—When currency is worth 80 eents on the dollar, \$500 in gold are worth how many dollars in currency?

Statement.—$\frac{100}{80} \times \frac{500}{1} = \frac{5}{4} \times 500 =$ \$625.00.

Solution.—Since one dollar in gold is worth $\frac{100}{80}$ or $\frac{5}{4}$ of a dollar in currency, 500 dollars in gold is worth 500 times $\frac{5}{4}$, or 625 dollars in currency.

Example 3.—100 dollars in gold is worth how many dollars in currency, when currency is worth $88\frac{8}{9}$ cents on the dollar?

Statement.—$\frac{100}{88\frac{8}{9}} \times \frac{100}{1} = \frac{9}{8} \times 100 =$ \$112.50 in currency.

Solution.—Since one dollar of gold is worth $\frac{100}{88\frac{8}{9}}$, or $\frac{9}{8}$ of a dollar in currency, 100 dollars of gold is worth 100 times $\frac{9}{8}$, or \$112.50 in currency.

Example 4.—What is 760 dollars in gold worth in currency, when currency is worth 95 cents on the dollar?

Statement.—$\frac{100}{95} \times \frac{760}{1} = \frac{20}{19} \times 760 =$ \$800.00 in currency.

Solution.—Since one dollar in gold is worth $\frac{100}{95}$, or $\frac{20}{19}$ of a dollar in currency, \$760 in gold are worth 760 times $\frac{20}{19}$, or \$800.00 in currency.

ANALYSIS OF MEASURING CORD WOOD.

128 cub. ft., or a pile of wood 4 feet high, 4 feet wide and 8 feet long is one cord of wood.

Example 1.—How many cords of wood in a pile 96 feet long, 4 feet wide and 8 feet high?

Statement. — $\frac{96 \times 4 \times 8}{8 \times 4 \times 4} = 12 \times 2 = 24$ cords.

Solution. — The product of the length, width and height divided by the factors that produces one cord; simply rejecting factors common to dividend and divisor, and we have 12×2, or 24 cords. Hence the rule:

Take the factors in one cord for the denominator of a fraction, and the factors in the pile of wood for the numerator of the fraction, perform the operations indicated, by rejecting factors common to numerator and denominator.

Example 2.—How many cords of bark in a pile 100 feet long, 6 feet high and 12 feet wide?

Statement.—$\frac{100 \times 6 \times 12}{4 \times 4 \times 8} = \frac{25 \times 6 \times 3}{8} = \frac{450}{8}$ $= 56\frac{1}{4}$ cords.

Solution.—The product of the factors in the pile of bark, divided by the factors of one cord gives $56\frac{1}{4}$ cords.

Example 3.—How many cords of wood in a pile 360 feet long 20 feet wide and 8 feet high?

Statement.—$\frac{360 \times 20 \times 8}{4 \times 4 \times 8} = 90 \times 5 = 450$ cds.

Solution.—Rejecting factors common to divisor and dividend, and we have $90 \times 5 = 450$ cords.

ANALYSIS OF MEASURING STONE.

There are $24\frac{3}{4}$ cubic feet in one perch of stone, because it is one and a half feet wide and one rod long, $16\frac{1}{2} \times \frac{3}{2} = 24\frac{3}{4}$, hence to find the number of perches in any number of cubic feet, simply divide by $24\frac{3}{4}$.

RULE FOR DIVIDING BY $24\frac{3}{4}$.

Remove the decimal point two places to the left in the number of cubic feet and multiply by four, and add one-hundredth part of the product for the business answer in all examples.

Example 1.—How many perches of stone in 1200 cubic feet?

Solution.—12.00
4
———
48, and 48 increased by the hundredth part of 48 gives 48.48 perches, answer.

Example 2.—How many perches of stone in 1300 cubic feet?

Removing the decimal point two places, we have 13, and $4 \times 13 = 52$, 52 plus $\frac{1}{100} \times 52 = 52.52$, ans.

Example 3.—How many perches in 1000 cubic feet of stone?

Removing the point two places we have 10, and $4 \times 10 = 40$, 40 plus $\frac{1}{100} \times 40 = 4.4$ perches.

All examples are performed the same. The number of cubic feet is found by multiplying length, width and height together, when the shape is regular. When in the form of a cone multiply the surface of the base by one third of the height. When in form of the frustrum of a cone add the upper base, lower base and the mean base together, and multiply their sum by one third of the height.

Example 4.—How many cubic feet of marble in

a block 24 feet long, 8 inches square at the base and 4 inches square at the top?

Solution.—$8 \times 8 = 64$ inches surface of the base, $4 \times 4 = 16$ inches surface of the top, the square root of $64 \times 16 = 32$ inches surface of the mean. The sum of $32 + 16 + 64 = 112$ inches, or $\frac{112}{144}$ or $\frac{7}{9}$ of a square foot, and $\frac{7}{9} \times \frac{1}{3} \times 24 = \frac{56}{9} = 6\frac{2}{9}$ cubic feet.

All examples of a similar character of stone, marble and wood are performed by the same rule.

Example 5.—How many feet of board measure in a stick of timber 36 feet long, 8 inches square at one end, and 4 at the other end?

Solution.—

$$\begin{array}{r} 8 \times 8 = 64 \\ 4 \times 4 = 16 \\ \sqrt{16 \times 64} = 32 \\ \hline 112 \end{array}$$

$$\frac{112}{144} = \frac{7}{9} \text{ of a square foot, and}$$

$\frac{7}{9} \times \frac{1}{3} \times 36 = \frac{84}{9} = 9\frac{1}{3}$ cubic feet, and since one cubic foot makes 12 feet of board measure, we have $12 \times 9\frac{1}{3} = 112$ feet board measure.

MEASURING CORN IN THE BIN, OR GRAIN OF ANY KIND.

RULE.—Remove the decimal point one place to the left in the number of cubic feet, and multiply by 8, and add $4\frac{1}{2}$ bushels to the result for each thousand; because $\frac{8}{10}$ of the number of cubic feet is nearly the number of bushels. The above rule is explained in the first part of this work. To measure the corn on the ear remove the decimal point one place to the left and divide by two and multiply by nine. In a crib of 3200 cubic feet how many bushels?

Solution.—

$$\begin{array}{l} 2)\underline{320.0} \\ \quad 160 \times 9 = 1440 \text{ bushels.} \end{array}$$

INTEREST ANALYSIS.

RULE FOR ALL RATES, ALL SUMS OF MONEY, AND ALL PERIODS OF TIME.

RULE.—*Invert the rate, annex ciphers and prefix points.*

The rule establishes the periods of time it takes a dollar to earn a cent, ten cents, one hundred cents and a mill: also the number of dollars it takes to earn a cent, ten cents, one hundred cents and a mill in one day, at any given rate per month or per annum. Hence all examples in interest at any rate are calculated for four business periods of time; also four corresponding sums of money for all periods of time. The above rule is illustrated and demonstrated in the first part of this book, also more thoroughly explained on pages from ninty-five to one hundred. The young student having perused carefully the pages referred to, is prepared to state in a few seconds any problem in simple interest in a form that it can be performed by a mere child understandingly, with ease and rapidity.

RULE FOR THE STATEMENT OF ANY EXAMPLE IN SIMPLE INTEREST.

RULE.—Take the period of time it takes a dollar at the given rate to earn a cent, ten cents, one hundred cents, or a mill for the first term of a proportion, the given time the second term, the principal the third term, the required interest is the fourth term; and is the product of the second and

third divided by the first. The statement may be written in the form of a fraction, indicating the product of the second and third terms by the sign of multiplication for the numerator of the fraction and the first term of the proportion for the denominator of the fraction. Rejecting common factors, if any, and performing the work indicated, and the result is in mills when the denominator is the time at the given rate, it takes a dollar to earn a mill, and in cents when the denominator is the time it takes a dollar to earn a cent, and in dimes when it is the time it takes a dollar to earn a dime. Hence point off three places to the left to reduce to dollars, when the result is mills, and two places when cents, and one place when dimes, &c.

Per annum 4 per cent. By the rule we have in 9 days a dollar earns a mill, and in 3 months a cent, and in 30 months a dime.

Illustration.—What is the interest of $231.00 for 27 days at 4 per cent?

Statement.—$\frac{\$231 \times 27}{9} = 231 \times 3 = 693$ mills, or $0.69,3, ans.

What is the interest of $732.00 for 15 months at 4 per cent?

Statement.—$\frac{\$732 \times 15}{3} = \$36.60$, answer.

Solution.—In three months a dollar earns a cent; hence 732 dollars earn in 15 months $\frac{732 \times 15}{3}$ cents, or $36.60.

All rates and all examples are handled by the same method.

Since our system of notation is founded on a base of ten, and the unity of our money the same base, ten mills making one cent, ten cents the dime and ten dimes the dollar; there can be no question but that the best practical method of computing interest is the *decimal method*. For any one understanding the decimal rule in this book can calculate the interest of a note easier and quicker than he can find it from an interest table.

Example 1. What is the interest of $500 for 123 days at $7\frac{3}{10}$ per cent per annum considering 365 days a year?—

The rule tells in a moment that $500 earn a dime in a day, hence in 123 days 123 dimes or $12.30. In 93 days it earns $9.30, and in 33 days, $3.30, in 60 days, $6.00 &c. All possible periods of time being calculated in a moment, by removing the decimal point one place to the left in the number of days, and reading the result dollars and the decimal part of a dollar. Now by the same law you observe that $50 earn a cent a day and you simply remove the decimal point two places to the left in the number of days and you have the interest in dollars and the decimal of a dollar. Also $5 earn a mill in a day, hence you remove the decimal point three places to the left in the number of days, and the result is the interest in dollars, cents and mills for any possible period of time you can name. Now by the same law $5000 you perceive, earn one dollar a day at $7\frac{3}{10}$ per cent per annum. Hence you read the number of days in dollars, or

simply prefix the sign of dollars to the number of days, and you have the interest. Now you can read and see clearly that time is money if you have perused the demonstration of the rule of interest, and the sequence of the rule found on page 99 of this book.

You observe by the rule *inverting the rate, annexing ciphers, and perfixing the decimal point,* give the *complete analysis of time,* for *any possible rate* per month, or per annum: establishing the periods of time in a moment that it takes a dollar to earn a mill, a cent a dime, and a dollar. Thus $7\frac{8}{10}$ per cent per annum, inverting, we have according to the rule; $\frac{1000}{78}$, hence a dollar earns a cent in 50 days, a dime in 500 days, a dollar in 5000 days, a mill in 5 days, a tenth of a mill in five tenths of a day &c.

Hence when a dollar earns a cent you may remove the decimal point two places to the left in any sum of money, for one cent is the hundredth part of a dollar, one dime the tenth part, one mill the thousandth and one tenth of a mill one tenthousandth part of a dollar, the rule giving you the time at any rate per cent it takes a dollar to earn a mill, cent, dime and dollar, you can calculate all possible sums of money for the four business periods of time quicker than you can find the interest of one sum from an interest table, or by the ordinary rule taught in our common schools.

Thus at $7\frac{8}{10}$ per cent, presume that millions of

examples are written, according to the law of writing numbers, dollars under dollars and cents under cents; thus removing the decimal point, four places to the left, or striking a line which represents the decimal point gives the interest for half a day. Now striking a line three places to the left, gives the interest for five days, two places for fifty, and one place to the left for 500 days, 5000 days, the note is the interest, and the decimal point remains unchanged. Now you must also observe that for sums of money corresponding to the periods of time established by the rule, you can remove the decimal point in all possible periods of time: thus a world of work is accomplished in less time than it takes to perform one example by the ordinary crooked and stupid way.

$34	7250.75
12	3435.68
1	3456.75
9	9999.25
	1000.50
	386.96
1	3456.12
	&c. &c.

PERCENTAGE.

Per cent means by the hundred, and the character $^0/_0$ is used as a substitute for the words per cent. Sometimes Percentage is a charge, or allowance of a certain number of units on every hundred in a given number, or quantity. Thus 1 per cent of $500 is $5, found by removing the decimal point two places to the left, to find 2 per cent multiply by 2, to find 3 per cent remove the decimal

point two places and multiply by 3 &c., for any per cent. The reason is obvious for the unity term is divided into 100 equal parts and the per cent represents the unit term: Thus 1 per cent means $\frac{1}{100}$, 2 per cent $\frac{2}{100}$, 3 per cent $\frac{3}{100}$, 4 per cent $\frac{4}{100}$ &c. The per cent taking the units term of a fraction, and 100 the unity term. to find $\frac{1}{2}$ per cent remove the decimal point two places to the left and divide by 2, to find $\frac{1}{3}$ per cent divide by 3 &c., for any fraction of a per cent.

Percentage is very easy to calculate, and useful; for a large portion of our business transactions are based on percentage. The solution of all questions, and all rules of percentage, like interest, is founded on the unity term and unit term.

Example 1. — A man buys a lot of goods, the price of which is $325, by paying cash he gets them 20 per cent off. What did he pay?

Solution. — 20 per cent off leaves 80 per cent to pay, or $\frac{80}{100}=\frac{8}{10}$, hence removing the decimal point one place to the left in $325 gives 32.50 and multiplying by 8 gives $260.00, what he has to pay. Reasoning from $\frac{1}{1}$ or the equivalent $\frac{100}{100}$ you get the answer easily and briefly, 10 per cent off, leaves 90 per cent to pay, or $\frac{9}{10}$, hence remove the decimal point one place to the left and multiply by 9 in all examples, 20 per cent off, leaves 80 to pay, or $\frac{8}{10}$, hence remove the decimal point one place to the left and multiply by 8 in all examples, 30 per cent off, leaves 70 per cent to pay, or $\frac{7}{10}$, hence remove

the decimal point one place to the left and multiply by 7 in all examples, 40 per cent off, leaves 60 per cent to pay, or $\frac{6}{10}$, hence remove the decimal point one place to the left and multiply by 6 in all examples.

50 per cent off, leaves 50 per cent to pay, or $\frac{5}{10}$ hence remove the decimal point one place to the left and multiply by 5, or divide the price of the goods by 2.

60 per cent off, leaves 40 per cent to pay, or $\frac{4}{10}$, hence remove the decimal point one place to the left in the price and multiply by 4.

70 per cent off, leaves 30 per cent to pay, or $\frac{3}{10}$, hence remove the decimal point one place to the left in the price and multiply by 3 in all examples.

80 per cent off, leaves 20 per cent to pay, or $\frac{2}{10}$. hence remove the decimal point one place to the left and multiply by 4 in all examples.

90 per cent off, a bill of goods leaves 10 per cent, or $\frac{1}{10}$ to pay, hence remove the decimal point one place to the left. The same principle applies to all possible per cents.

The price of a lot of goods is $50, by paying cash you get them 19 per cent off. What do you pay for them; 19 per cent off leaves 81 per cent to pay or $\frac{81}{100}$, hence remove the decimal point two places to the left in $50, gives $.50 and multiplying by 81, gives $40.50 what they cost.

After purchasing goods, what must they be sold for by the article, to make a certain per cent more than the cost.

Example. — A man buys hats at $8. per dozen, what must he sell them a piece to make 20 per cent? They cost $\frac{1}{1}$ or $\frac{5}{5}$ and to make 20 per cent, or $\frac{1}{5}$, he must sell them for $\frac{5}{5}+\frac{1}{5}$, or $\frac{6}{5}$ of what they cost, and the selling price of one hat to make 20 per cent, is $\frac{1}{12}$ of $\frac{6}{5}=\frac{6}{60}$ or $\frac{1}{10}$ hence to make 20 per cent on all articles bought by the dozen, and sold by the piece, remove the decimal point one place to the left in the price per dozen. The solution of all other per cents are similar.

Example. — A man sells a horse for $150 and makes 25 per cent, he then sells another horse for $150 and looses 25 per cent. Does he make or lose by the two operations, and how much?

Solution.— Let $\frac{1}{1}$ represent what the horse cost; first sale he made 25 per cent or $\frac{1}{4}$ the cost $\frac{1}{1}+\frac{1}{4}=\frac{5}{4}$ what he sold him for, hence $150 is $\frac{5}{4}$, and $\frac{1}{4}$ of what the horse cost is $\frac{1}{5}$ of $150, or $30, and $\frac{4}{4}$, or $\frac{1}{1}$ the cost of the first horse is 4×$30 $120 hence he made on the first sale $150—$120, or $30.

The second horse he lost 25 per cent, or $\frac{1}{4}$, second horse cost $\frac{1}{1}$, and he lost $\frac{1}{4}$, he must have sold him for $\frac{3}{4}$ of what he cost, hence $150 is $\frac{3}{4}$, and $\frac{1}{4}$ is $\frac{1}{3}$ of $150, or $50 and $\frac{4}{4}$ or $\frac{1}{1}$ is four times $50, or $200, hence on the second sale he lost $200—$150, or $50. On the first sale he made $30, second sale lost $50, hence he lost $20.

PRESENT WORTH AND DISCOUNT.

Discount is an allowance made for the payment of a debt before it becomes due.

Commercial Discount is a deduction from the nominal price of an article.

Bank discount is interest paid in advance, and for 3 days more than the nominal time, called Days of Grace.

The Proceeds of a note given at a bank is the amount which the bank pays for the note.

True Discount is a deduction made for the present payment of a sum of money due at some future time.

The Present Worth of a sum of money due at some future time, is a sum which, put at interest at a rate agreed upon, will in the given time amount to the sum due.

Present Worth and Discount, present no special difficulties, when you reason from the unity term in stating the problem. Thus the present worth of any sum of money due at a future time, is found by multiplying the debt by the amount of one dollar for the given time, at the given rate, taken as the unity term of a fraction and one hundred, the number of cents in a dollar for the unit term of the fraction.

What is the present worth of a debt of $1650 without interest, due 8 months hence, money being worth 6 per cent.

Statement. — 1650\times\frac{100}{104}$, rejecting the common factors, we have $1586.53 for the present worth. For the true discount we subtract $1586.53 from $1650 which gives $63.47, the true discount.

Analysis and reason of Statement.

The interest of $1. for 8 months at 6 per cent per annum is four cents, hence $1. amounts to 104 cents, in 8 months, and the present value of one dollar is as much as 104 is contained in 100, the number of cents in a dollar as indicated in the statement, hence multiply by the number of dollars in the debt.

RULE.— Multiply the debt by 100 divided by the amount of $1. for the given time, at the given rate, the result is the present worth. Subtracting the present worth from the given sum, the remainder is the true discount.

What is the present worth of a debt of $330, due 15 months hence, money being worth 8 per cent?

Statement. — 330\times\frac{100}{110}$, performing the operation indicated we get $300 for the present worth.

What is the present worth of a debt of $880 due 30 months hence, money being worth 4 per cent?

Statement.— 880\times\frac{100}{110}$=$800 the present worth.

All examples are stated precisely the same, simply take the amount of one dollar, at the given rate, for the given time, for the unity term of the fraction, and 100 for the unit term of the same fraction and write the sign of multiplication between that expression and the given sum of mo-

ney and the problem is stated, performing the operations indicated by the statement and you have the answer.

RULE.—The discount is found by multiplying the interest of the given sum for the specified time and rate by 100 divided by the amount of one dollar for the given time at the given rate.

Thus in the last example the interest of $880 due 30 months hence at 4 per cent is $88, and $88× $\frac{100}{110}$=$80 the true discount.

All examples in present worth and discount are stated and performed by the preceeding two brief rules.

STOCKS AND BONDS.

A Company is an association of persons for transacting business. A Business Corporation is an association authorized by special or general law to transact certain business, under a specified name.

A Firm is an association bound to each other by mutual articles of agreement for the transaction of certain business.

Capital Stock is the amount of money paid, together with that subscribed, for the purpose of carrying on the business of the company. Stocks are the certificates of a corporation signed by the proper officers showing that the holder owns so many shares in the capital stock of the company, any one who owns stocks is a stockholder in the company.

INSURANCE.

Insurance is a contract by which one party, in consideration of a certain sum of money paid, engages to indemnify another for a loss which he may sustain by certain casualties.

The Insurer is the party who makes the contract and takes the risk.

The Premium is the sum paid for the Insurance.

The Policy is the written contract made by the insurer.

The rule of Percentage applies in computing Insurance.

STATE AND LOCAL TAXES.

A Tax is a sum of money assessed upon the person or property of an individual for the support of the government and other public purposes..

A Poll Tax is a tax levied upon the person of each male citizen liable to pay taxes, without regard to property.

Taxable Property is either Real or Personal, Real Property consists of fixed property, lands and houses.

Personal Property consists of moveable property, cash, stocks, ships &c.

RULE.—To find the rate of property-tax, make the value of the taxable property the unity term of a fraction, and the sum to be raised, minus the amount assessed on the polls, the unit term of the fraction.

To find each persons tax, multiply his taxable property by the rate, and to the product add his poll-tax.

Having found the tax on $1.00 you can facilitate the calculation by preparing a tax table thus:

Table at the rate of 4 mills on the Dollar.

PROPERTY	TAX.
$1	$0.004
2	.008
3	.012
4	.016
5	.02
6	.024
7	.028
8	.032
9	.036
10	.04

Now by removing the decimal point to the right as the case requires, the tax is found on any sum of money.

CUSTOM-HOUSE BUSINESS.

Custom-Houses are branches of the Treasury Department, established by the General Government for the collection of duties, each being controlled by a Collector and Naval Officer, who are responsible for such collection to the Secretary of the Treasury.

Duties are Specific, Advalorem or Combined, a Specific Duty is a rate of duty chargeable upon quantity, without regard to cost, an advalorem Duty is a rate of duty, chargeable upon the value of the goods at the last port of exportation.

A Combined Duty is a combination of advalorem and specific duties.

The Gross Weight is the weight of the goods with what contains them.

The Net Weight is the weight of the goods after all deductions have been made.

The Dutiable Value of Merchandise under the present law, is the original cost, or wholesale price.

INTEREST ON ENGLISH MONEY.

Interest is calculated on a basis of 365 days to the year in England.

Twelve months are usually reckoned a year. For days, such a part of one years interest is taken as the number is of 365.

To compute interest on English Money. The rule that I have given for United States Money applies most beautifully. That rule and its sequence is all you need to understand.

Inverting the rate per cent and annexing ciphers and prefixing the decimal point to the rate inverted, establishes the periods of time that it takes £1 to earn a $\frac{1}{1000}$, a $\frac{1}{100}$, a $\frac{1}{10}$ of a pound and £1, in all possible rates of interest. The demonstration is similar to that given for United States Money, hence it is superfluous to repeat it here, I will give only one illustration.

Presume the rate of interest to be $7\frac{3}{10}$ per annum, that means £100 earns £$7\frac{3}{10}$ in one year.

Now by the rule inverting the rate and annexing ciphers, and prefixing points we have this ex-

pression, $^{1}\frac{00}{78}{}^{0}$ which translated into the concrete reads in half a day £1 earns $\frac{1}{10000}$ of a pound, 5 days $\frac{1}{1000}$, 50 days $\frac{1}{100}$, 500 days $\frac{1}{10}$ and 5000 days £1 earns £1. Hence it is easy to understand that all examples are calculated for the five business periods of time without changing or making a figure, only removing the decimal point.

Now presume all the examples that ever occurred in England were written according to the law of writing them, the same denominations under each other.

Were shillings, pence and farthings added to each note, you simply remove the point on them the same as the pounds. Now observe that all periods of time are calculated for the sums of money corresponding to the periods of time established by the rule.

Thus in the example £1 at $7\frac{3}{10}$ per cent earns £$_{1}\frac{1}{0}$ in 500 days, hence £500 earns £$\frac{1}{10}$ in one day, hence all time is calculated for that sum.

	½ day.	5 days.	50 days.	500 days.
£34	7	6	8	5
	2	7	6	8
1	2	3	4	6
5	6	8	3	8
123	4	6	8	9
999	9	9	9	9
33333	3	3	3	3
1	0	0	0	0
		1	0	0
			2	2

RULE. — For ½ day remove the point four places to the left, five days three places, fifty days two places, fivehundred days one place, five thousand days the note is the interest and the point is unchanged.

Illustration. What is the interest of £500 at $7\frac{3}{10}$ per cent per annum for all possiblе periods of time that have occurred, or can occur.

Now according to my universal law of calculating the interest for all periods of time, and all sums of money, and at any per cent.

Thus £500 at $7\frac{3}{10}$ per cent per annum for days, removing the decimal point one place to the left in the number of days gives the interest of £500 at $7\frac{3}{10}$ per cent per annum, in pounds and the decimal of a pound, hence all periods of time that have occurred in England, or any other country, or may occur are calculated by one lightning stroke of the pen. Now £50, the interest is found by removing the decimal point two places to the left in the number of days, £5 three places to the left, 10s. four places to the left 1s. five places to the left, and £5000, the point remains unchanged, the number of days being the interest in pounds, hence you perceive that time is *really* money.

£36500000	0
25000	0
177	6
187	8
36	5
36	0
3	0
3	3
9	0
9	3
&c. &c.	

ANNUAL INTEREST.

Annual Interest is simply interest on the principal and on the interest over due on promissory notes, or other contracts containing the words,

with annual interest. Interest upon the interest on such contracts is allowed by the courts in some States, in the return of damages for the non payment of the interest when it falls due, otherwise it cannot be considered legal.

What will $4000 amount to in 10 years and 6 months, annual interest at 6 per cent?

Solution. The interest on $4000 at 6 per cent for one year is $240, for $10\frac{1}{2}$ years it is $\$240\times10\frac{1}{2}=$ $2520.

The interest of $240 for one year is $14.40. $9+8+7+6+5+4+3+2+1\frac{1}{2}=45\frac{1}{2}$ years and multiplying $14.40 by $45\frac{1}{2}$, gives $655.20.

Hence $4000
2520
655.20

$7175.20 is the amount for $10\frac{1}{2}$ years.

COMPOUND INTEREST.

Compound Interest is calculated upon the principal and interest added together annually, semi-annually &c.

RULE.— Multiply the principal by the amount of $1 at the given rate raised to a power equal to the number of years the note is on interest, the result is the amount, subtract the principal from the amount and you have the compound interest.

What is the compound interest of $1000 for 3 years at 6 per cent?

The amount is $\$1000\times(1.06)^3=\$1191.016$, and $\$1191.016-\$1000=\$191.016$ the compound interest.

Dividing 72 by the rate gives about the time it takes a note to double at compound interest at any rate from 2 to 10 per cent.

MEASUREMENT OF LUMBER.

To find the superficial contents of a board one inch thick, when the length is given in feet and the breadth in inches.

RULE.—Take 12 for the unity term of a fraction, and the length and width united by the symbol $\times$ for the units term performing the operation indicated and you have the answer.

Example 1. — How many square feet are there in a board 24 feet long and 9 inches in width?

Statement. — $\frac{24\times9}{12}=18$ feet simply rejecting the common factor, 12 and multiplying by 9, the other factor in the statement.

Example 2. — How many square feet in a board 30 feet long and 16 inches wide?

Statement. — $\frac{30\times16}{12}=40$ feet. Dividing the unity term and unit term by 3, and then by 4, we have 10×4 or 40.

Example 3. — How many feet in a board 18 feet long and 16 inches wide?

Statement. — $\frac{18\times16}{12}=24$ feet, simply $\frac{4}{3}\times18=24$ answer, or $\frac{3}{2}\times16=24$.

Example 4.—How many feet in ten boards, each 20 feet long and 24 inches wide.

Statement. — $\frac{20 \times 24 \times 10}{12} = 400$ feet.

The 12 is contained in 24, 2 times, hence $20 \times 2 \times 10 = 400$ the number of feet.

Example 5. — How many feet in 500 boards, each board 16 feet long and 8 inches wide.

Statement. — $\frac{16 \times 8 \times 500}{12} = 5333\frac{1}{3}$.

Example 6. — How many feet in 1200 boards, each 18 feet long and 16 inches wide?

Statement. —

$$\frac{18 \times 16 \times 1200}{12} = 18 \times 16 \times 100 = 28800 \text{ feet.}$$

Now were the boards two inches in thickness, it is obvious they would contain twice as much lumber, 3 inches, three times, and one half an inch half as much &c.

You observe that you get the same answer as you would by finding the number of square inches in the board, and dividing by 144 the number of square inches in one square foot.

Thus a board 16 feet long 18 inches wide contains $\frac{16 \times 18}{12}$, which equals 24, amounts to the same as $\frac{12 \times 16 \times 18}{12 \times 12} = 24$ and is a much shorter method.

TO FIND THE CONTENTS OF PLANKS, SCANTLINGS, JOISTS AND SQUARE TIMBER.

1. Find the contents in board measure of a plank 16 feet long, 8 inches wide, and 2 inches thick.

Statement. — $\frac{16 \times 8 \times 2}{12} = \frac{64}{3} = 21\frac{1}{3}$ feet.

2. Find the contents of a plank 14 feet long, 10 inches wide and 3 inches thick.

Statement. — $\frac{10\times14\times3}{12}=35$ feet.

3. How many feet in 12 pieces of scantling, each 14 feet long, 3 inches wide and 2 inches thick?

Statement.—$\frac{14\times3\times2\times12}{12}=84$ feet of board measure.

4. How many square feet of lumber in 100 pieces of scantling, 3 by 2 and 15 feet long?

$$\frac{100\times15\times3\times2}{12}=750 \text{ feet.}$$

5. Find the contents of a stick of timber 8 by 4, and 15 feet long, $\frac{8\times4\times15}{12}=40$ feet.

6. How many feet are there in 1000 pieces of scantling $3\frac{1}{2}$ by $2\frac{1}{2}$ and 16 feet long?

$$\frac{1000}{1}\times\frac{7}{2}\times\frac{5}{2}\times\frac{16}{12}=\frac{35000}{3}=11666\tfrac{2}{3} \text{ feet.}$$

SQUARE AND ROUND TIMBER.

To find the cubical contents of square timber.

RULE.—Take 12×12 for the unity term of a fraction and the indicated product of the area of one end in inches and the length in feet for the unit term.

What are the cubical contents of a stick of timber 16 inches square, and 20 feet long.

Statement. — $\frac{16\times16\times20}{12\times12}=\frac{320}{9}=35\frac{5}{9}$.

Find the cubical contents of a stick of timber 16×18, and 36 feet long.

Statement. — $\frac{16\times18\times36}{12\times12}=72$ feet.

For round timber to reduce it to square timber

RULE.—From the mean diameter subtract its third part, square the remainder, and the product of that result into the length divided by 12×12 gives the cubical contents in square timber.

NOTE. To find the mean diameter, add the two ends together and divide by two.

EQUATION OF PAYMENTS AND AVERAGING ACCOUNTS.

Equation of Payments is the process of finding the average time for the payment of several obligations due at different dates.

The method of averaging accounts is based upon the principle that the use of any sum of money paid before it is due, is equivalent to the use of an equal sum for the same length of time after it becomes due.

Illustration.—Suppose A owes B $500 due in one year, and $500 due in two years, without interest. What is the average maturity of both debts? 18 months from the date of the first is the average maturity of both sums; because A will then have had the use of $500 for six months after it became due, which is equal to the use of $500 paid six months before it is due.

Rule covering all cases of Equation of Payments and Averaging Accounts.

RULE.—Unite in one sum, at any rate per cent when none is named, the interest on each obliga-

tion from its maturity to the most remote maturity, making the result the unit term of a fraction, and the interest on the sum of the obligations at the same rate the unity term of the same fraction. Subtract this time from the date of the most remote maturity, or add as the case may require.

The Equated Time is the date when there is an equilibrium of interest between the two sides, so the balance of the account is then due without interest.

Example. — A owes B January 1st 1878, $1800; of which $700 is payable in 6 months; $300 in 4 months, and $800 in 18 months, when can the whole be paid without gain or loss of interest to either party?

Now by the rule we have for the unit term of the fraction, the interest of $700 for 12 months, and $300 for 14 months, which is $63, allowing the rate to be 6 per cent, and for the unity term of the fraction the interest of $1800 at 6 per cent for one month which is $9. Hence $\frac{63}{9}$ or 7 months before the expiration of the 18 months, or December 1st 1878, for the equated time.

My interest rule and time table make Equation of payments and Averaging accounts a very easy calculation.

TO FIND THE SUM OF A SERIES OF TERMS, INCREASING BY A COMMON DIFFERENCE.

The fundamental rule for all examples is, take two for the unity term, and the first term plus the last term for the unit term then multiply the expression by the number of terms. This plain rule gives the sum of any series of numbers increasing by a common difference.

Example 1. — How many times does a clock strike in eleven hours?

Statement. — $\frac{1+11}{2}\times 11 = 66$ ans.

Example 2. — How many times does a clock strike in seven hours?

Statement. — $\frac{1+7}{2}\times 7 = 28$ ans.

Example 3.— How many cannon balls in a pile the form of a pyramid, there being 49 in the bottom row, and one on top?

Statement. — $\frac{49+1}{2}\times 49 = 1225$ ans.

Example 4. — How many shot in a pyramid, 9999 in the bottom row and one on top?

Statement. — $\frac{9999+1}{2}\times 9999 = 49995000$ ans.

NOTE. The same rule applies, whatever the common difference may be.

TO FIND THE SUM OF A SERIES OF TERMS INCREASING BY A CONSTANT MULTIPLIER.

The universal rule for all examples is: Take the multiplier minus one for the unity term, and the product of the last term and multiplier diminished by the first term for the unit term. The rule to find the last term is multiply the constant multiplier raised to a power one less than the number of terms by the first term and you have the last term of the series.

Example. — A man agrees to work for 32 days on the condition that he gets one cent the first day, two the second, 4 the third, &c., each days wages increasing by the constant multiplier two. What is the amount of the 32 days wages?

Statement. — $\dfrac{2^{32} - 1}{2 - 1}$

Operation. —

$$
\begin{array}{r}
4 = 2^2 \\
16 = 2^4 \\
{\scriptstyle 2\,5\,6\,3} \\
\hline
256 = 2^8 \\
{\scriptstyle 6\;\;9\,9\,13\,9\,7}\;256 \\
\hline
65536 = 2^{16} \\
65536 \\
\hline
\$42949672.96 - 1 = 2^{32} - 1
\end{array}
$$

My Universal Law of multiplication makes all examples under these rules brief.

TO FIND THE SUM OF A SERIES OF TERMS DECREASING TO INFINITY BY A CONSTANT MULTIPLIER.

RULE. — Take one minus the multiplier for the unity term and the first term of the series for the unit term performing the operation indicated by the statement and you have the sum of the decreasing series.

Example 1. — What is the sum of $1+\frac{1}{3}+\frac{1}{9}$ &c. to infinity?

Statement. — $\dfrac{1}{\frac{2}{3}} = 1\frac{1}{2}$ ans.

Example 2. — What is the sum of

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$ &c., to infinity?

Statement. — $\dfrac{1}{\frac{1}{2}} = 2$ ans

Example 3.—What is the sum of $8+2+\frac{1}{2}+\frac{1}{8}$ &c., to infinity?

Statement. — $\dfrac{8}{\frac{3}{4}} = 10\frac{2}{3}$ ans.

NOTE. Eight is the first term of the series in the last example, ¼ is the multiplier producing the terms.

The preceding rules are so plain and so easily applied it is not necessary to give any more examples; except perhaps in calculating compound interest.

TO DISCHARGE A DEBT OF PRINCIPAL AND INTEREST IN A GIVEN NUMBER OF EQUAL ANNNUAL PAYMENTS.

RULE. Take the compound interest of one dollar, for the given time, at the given rate for the unity term, and the interest of the principal for one year multiplied by the amount of one dollar for the given number of years for the unit term, performing the operation indicated in the statement and you have one of the equal payments.

Example 1.—What must be one of the equal an nual payments which will discharge a 10 per cent note for $6000 in 5 years?

Statement. — $\frac{1.61051\times600}{.61051}=\$1582.78$, which is one of the equal payments.

NOTE. To discharge a $2000 debt same rate and time take one third of the above answer, for $1200 one fifth, &c.

By the preceding rules the labor is very much abreviated, the unity term and unit term are found by the rule for finding the sum of a series of terms increasing by a constant multiplier.

MEASURING LAND.

Since a chain is four rods long, a square chain must be 16 square rods, and 10 square chains 160 square rods, or one acre. Hence the measuring unity of land is 10 square chains. And the rule

for all examples is: Take 10 for the unity term and the indicated product of length and width in chains for the unit term, performing the operation indicated in the statement and you have the number of acres.

Example 1. — How many acres in a field 12 chains long and 9 chains wide?

Statement. — $\frac{12 \times 9}{10} = 10.8$ acres.

Example 2. — How many acres in a plot of land 20.75 chains long and 20.25 chains wide?

Statement. — $\frac{20.75 \times 20.25}{10} = 42.01875$ the number of acres.

NOTE. 100 links make one chain, and part of a chain is written decimally.

MEASURING LAND WITH THE ROD POLE.

RULE. — Take for the unity term 4×40 and for the unit term the indicated product of length and width.

Example. — How many acres in a plot of land, 480 rods long and 120 rods wide.

Statement. — $\frac{480 \times 120}{4 \times 40} = 360$ acres.

Simply rejecting common factors. All examples are stated and performed the same way.

TO FIND THE AMOUNT OF SQUARE-EDGED INCH BOARD THAT CAN BE SAWED FROM A ROUND LOG.

RULE. Substracting four inches from the diameter in inches, and squaring the remainder, gives the amount of lumber in a log sixteen feet long, increasing or diminishing according to the law of ratio, taking sixteen as the measuring unity; the business answer is quickly found for all examples.

Example 1. — How much square-edged inch lumber can be cut from a log 20 inches in diameter, and 12 feet long?

Statement. — $\frac{12}{16} \times 16^2 = 12 \times 16 = 192$ ans.

Example 2. — How much square-edged inch lumber can be cut from a log 24 inches in diameter, and 18 feet long?

Statement. — $\frac{18}{16} \times 20 \times 20 = \frac{9}{8} \times 400 = 450$ feet.

You observe the 16 takes the place of the unity term of a fraction and the length of the log the place of the units term of a fraction, you substract four from the diameter in inches, square the remainder and multiply the result by the ratio, the length of the log bears to 16 the measuring unity.

This rule is the basis of the tables in Scribner's popular "Lumber and Log Book", which is a standard work among lumbermen.

TO ADD THE ALPHABET OF NUMBERS.

$$\tfrac{1}{1} \text{ plus } \tfrac{2}{1}+\tfrac{3}{1}+\tfrac{4}{1}+\tfrac{5}{1}+\tfrac{6}{1}+\tfrac{7}{1}+\tfrac{8}{1}+\tfrac{9}{1}+\tfrac{0}{1}=45$$

RULE FIRST. — Simply nine times one-half the ten digits.

RULE SECOND.— By taking three times the mean thus:

$$9+8+7+6+5+4+3+2+1=45;\text{ or}$$
$$3\times8+3\times5+3\times2=45,$$

the sum of three numbers is three times the mean. The sum of two numbers is twice their mean.

BUSINESS METHOD OF ADDITION.

RULE THIRD. — Commence at the right hand column; find the sum, which is 33, write the three tens over the column of tens and the three units under the column of units. Find the sum of the second column, which is 61, write the six over the column of hundreds, and the one under the column of tens, &c., or add it thus, striking a line down and out, cutting off all but the last figure to the left.

$$\begin{array}{r} {\scriptstyle 563} \\ 7351 \\ 6582 \\ 9164 \\ 3867 \\ 4932 \\ 8379 \\ 6183 \\ 4762 \\ 9351 \\ 8642 \\ \hline 69213 \end{array} \qquad \begin{array}{c|c} 3 & 3 \\ 6 & 1 \\ 5 & 2 \\ \hline 6 & 9 \end{array}$$

RULE FOURTH. — The lightning method of addition by combination, taking 18 as the base.

Rule. — Prefix the number of nines to the miscellaneous row, strike a line and subtract the number of nines.

$$\begin{array}{r} 7859 \\ 3986 \\ 8153 \\ 3124 \\ 9986 \\ 6888 \\ 4378 \\ 9634 \\ 5986 \\ \hline 6182\cancel{6} \\ 0 \end{array}$$

THE GENERAL LAW OF MULTIPLICATION.

The general law of multiplication must be established clearly and thoroughly in the mind, then all special, and lightning rules are quickly explained, and easily comprehended.

Hence I will present and apply it in three ways. First, the illustration, then, the rule or law finally the proof.

First we write the terms of the multiplier under the corresponding terms of the multiplicand, strike a line below the multiplier, and above the multiplicand, and you are ready to proceed in finding the product of any two numbers at once.

Thus what is the product of two thousand four hundred and forty three by three thousand one hundred and forty two? Writing thus:

$$\begin{array}{r} \underline{12322} \\ 2443 \\ \underline{3142} \\ 7675906 \end{array}$$

The figures above the line register, and fix the mental operation as you are proceeding in writing the answer. Hence the brain is not tired, or the mind confused for a moment by what is called the carrying figure.

Now commence at the left hand, and find the product, thus:

```
   2443
   3142
6443706
 12322
7675906
```

Now you may commence at the right hand, again and perform it thus:

```
   2443
   3142
1232206
64437
7675906
```

```
13 12 11 10 9 8 7 6 5 4 3 2 1
13 12 11 10 9 8 7 6 5 4 3 2 1
```

The numbers above indicate the number and the position of the corresponding terms in the multiplicand and multiplier, to get the general law of multiplication established in the mind. First take two terms by two, then three by three, and four by four &c. Applying the rule definitely as you proceed, thus:

```
  1
 23
 32
736
```

RULE 5. — The first term of the multiplicand by the first term of the multiplier, then the first by the second, and the second by the first, and the second by the second.

Now for three terms as;

```
 2 2 1
   4 2 3
   3 3 2
1 4 0 4 3 6
```

When you reach the third term, you multiply the third term of the multiplicand by the first term of the multiplier, and the second term by the second and the first by the third &c., for any number of figures in multiplicand and multiplier.

By performing two or three examples from right to left, and left to right you will fix the law of operation in the mind never to be forgotten. Thus:

```
1 2 3 4 6 7 6 7 5 4 4 1
          1243162
          3425346
    4258259984052
```

Rule 6.—Multiply the first term of the multiplicand by the first term of the multiplier, registering the carrying number over the second term, and the answer figure under the first term, then the second term by the first, and the first by the second, adding as you multiply, the products and the registered number, writing the carrying number over the third term and the answer figure under the second term, then the third term by the first, and the second by the second and the first by the third, adding as before, writing the carrying number over the fourth term, and the answer figure under the third, then the fourth term by the first, the third by the second, the second by the third, and the first by the fourth, writing the carrying number over the fifth term, and the answer figure under the fourth term; then the fifth term by the first, the fourth by the second, the third by the

third, the second by the fourth and the first by the fifth, writing the carrying number over the sixth term, and the answer figure under the fifth term; then the sixth term by the first, the fifth by the second, the fourth by the third, the third by the fourth, second by the fifth, and the first by the sixth, writing the carrying number over the seventh term, and the answer figure under the sixth term; then the seventh term by the first, the sixth by the second, the fifth by the third, the fourth by the fourth, the third by the fifth, the second by the sixth, and the first by the seventh, writing the carrying number in its proper place and the answer figure under the seventh term. When the carrying number is one place by the last term of the multiplicand, you drop the first term of each factor, then, as in the example given, you take the seventh term of the multiplicand by the second term of the multiplier, the sixth by the third, the fifth by the fourth, the fourth by the fifth, the third by the sixth and the second by the seventh writing the carrying number in its place and the answer figure in its place. Now drop the second term of each factor, and take the seventh by the third, the sixth by the fourth, the fifth by the fifth, the fourth by the sixth, and the third by the seventh; writing the carrying number in its place, and the answer figure in its place. Now drop the third term of each factor, and take the seventh term by the fourth, the sixth by the fifth, the fifth by the sixth, and the fourth by the seventh; writ-

ing the carrying number in its place, and the answer figure in its place. Now drop the fourth term of each factor, and take the seventh term by the fifth, the sixth by the sixth, and the fifth by the seventh, registering as usual; now drop the fifth term of each factor, and take the seventh term by the sixth, and the sixth by the seventh, adding the registered number as usual, and the seventh term by the seventh, writing the product. Proceeding by the same law of operation, the product of any number of terms in multiplier and multiplicand of Arithmetic and Algebra, is quickly found.

If you take one figure by one, then two by two, three by three and four by four, you will have no trouble in comprehending the law of operation, Thus:

	36	236	13236
7	27	127	2127
9	29	129	2129
63	783	16383	4528383

or take similar numbers first, thus:

	111	85785581
1111	2222	44444
1111	2222	44444
1234321	4937284	1975269136

282	224822	184481
234	3416	23473
342	3234	324
80028	11047344	7605252

11111111111111
1111111111111111
1111111111111111
1234567901234567654320987654321

Fundamental proof and the reason is presented clearly in the work. You simply multiply the excess of nines together of the two factors, and the excess of nines in that product must equal the excess of nines in the answer. As in the last example, the excess of nines in the multiplier is seven, also multiplicand seven, product 49, excess of nines in 49 is four, hence the excess of nines in the answer is four. One illustration will show you the philosophy of the proof.

The number

$$
\begin{array}{rr}
 & \underline{2_6 8_5 4_4 6_3 3_2 9_1 7_0} \\
= & 2\ 2\ 2\ 2\ 2\ 2\times9+2 \\
 & 8\ 8\ 8\ 8\ 8\times9+8 \\
 & 4\ 4\ 4\ 4\times9+4 \\
 & 6\ 6\ 6\times9+6 \\
 & 3\ 3\times9+3 \\
 & 9\times9+9 \\
 & 0\times9+7
\end{array}
$$

Note. — The numbers under the given number indicate the exponents of the base of numbers, the zero power, first power, second power, third power, fourth power, &c.

Any number can be written in its value of nines, thus:

$$
\begin{array}{lr}
2_6 = & 222222\times9+2 \\
8_5 = & 88888\times9+8 \\
4_4 = & 4444\times9+4 \\
6_3 = & 666\times9+5 \\
3_2 = & 33\times9+3 \\
9_1 = & 9\times9+9 \\
7_0 = & 0\times9+7
\end{array}
$$

Hence the proof is by retaining the nines instead of casting them out.

The rule of multiplication is readily acquired and easily remembered by referring to these figures:

2221 4321 4321 18671041	representing the first term, second term, third term, &c., of multiplier and multiplicand.

Rule.—1st term by the 1st, 1st by 2nd and 2nd by first, 1st by 3d 2nd by 2nd and 3rd by 1st, 1st by 4th 2nd by 3rd 3rd by 2nd and 4th by 1st, 2nd by 4th 3rd by 3rd and 4th by 2nd, 3rd by 4th 4th by 3rd, 4th by 4th, &c., for any number of terms.

Note.—You may reverse the multiplier, thus:

$$\begin{array}{r} {\scriptstyle 2221} \\ 4321 \times 4321 \\ \underline{1234} \\ 18671041 \end{array}$$

and each result in the answer is found without crossing, the mind moving in parallel lines to find each answer figure.

DIVISION BY COMPLEMENT AND SUPPLEMENT.

Complement means what it takes to complete, and supplement, surplus. 99 the complement is 1, it takes one to make the complete 100.

101 the supplement is 1, it is 1 more than 100.

First to divide any number by one figure.

Example 1. — How many times is 9 contained in 989164?

```
9| 9 8 9 1 6 4 | 109907 1/9
1|1.0.8 9
    9.8 1
     9.0.6 4
        7.1
```

Rule. — Write the divisor at the left of the dividend, and the complement of the divisor directly under the divisor, find how many times the divisor is contained in the first left hand part of the number, and multiply by the complement, and add it to the part of the number divided, without writing down only the sum and point off the left hand figure as you proceed, thus proving each figure and aiding to see what the next figure in the answer must be.

Example 2. — How many times is 8 contained in 987465?

```
8| 9 8 7 4 6 4 | 123433
2|1.1 8
   2.2 7
     3.3 4
       4.2 6
         3.2 4
           3.0
```

Note. — To find the complement of one figure subtract it from ten, two figures from hundred, three from 1000 &c.

Example 3. — How many times is 99 contained in 9867494?

```
99 | 9.8 6 7 4 9 4 | 99762 56/99
 1 |   9.6 6       |
        7.5 4
          6.1 9
            2.5 4
              5 6
```

Note.— Multiply the complement by each quotient figure and add and point off one.

Example 4. — How many times is 98 contained in 9876494?

```
98 |  9 8 7 6 4 9 4 | 100780 54/98
 2 | 1 0.0.7.6 4    |
           7 8 9
           8.0.5 4
```

Example 5.— How many times is 997 contained in 99865943256?

```
997 |  9 9 8 6 5 9 4 3 2 5 6 | 100166442 582/997
  3 | 1.0.0.1.6 5 9          |
           6.6 2 4
             6.4 2 3
               4.4 1 2
                 4.2 4 5
                   2.5 7 6
                     5 8 2
```

We will perform the same example by the usual method.

```
997|99865943256|100166442 582/997
   |997
    ----
     1659
      997
     -----
      6624
      5982
      -----
       6423
       5981
       -----
        4412
        3988
        -----
         4245
         3988
         -----
          2576
          1994
          -----
           582
```

Note.— The first method gives you the proof as you proceed and saves paper and mental labor, and is universal in its application to all numbers.

Note. — The complement of the divisor is found by subtracting it from the next higher unity. When the divisor contains one figure subtract from 10 to find the compliment, two figures from 100, three figures from 1000, four figures from 10000, &c.

The reason of the rule becomes evident by a moments reflection on the nature of division. The reason of multiplying the complement of the divisor by each quotient figure and adding instead of subtracting to find a new dividend, is because the complement is a negative quantity and the sign must be changed to subtract.

Hence it becomes positive and you add. Thus

100—2=98, the negative two is the complement of 98; 10—1=9, the minus 1 is the complement of 9; 1000—4=996, the minus 4 is the complement of 996, and the general law of subtraction is to change the sign of the subtrahend and add.

You can divide by multiplying each quotient figure by the supplement of the divisor, and subtracting from the dividend because the supplement is a positive number.

METHOD OF SQUARING NUMBERS BY COMPLEMENT AND SUPPLEMENT.

Rule Eight.—For Squaring a whole number or a fraction.

Increase the number by its supplement, multiply by the base, and add the square of the supplement; diminish the number by its complement, multiply by the base, and add the square of the complement.

Note.—Take the most convenient number for the base.

Rule Nine.—To multiply numbers.

To multiply two numbers, find their mean, square it, and subtract the square of half their difference.

Rule Ten.—When the sum of the units equal ten, and the tens are equal, increase the tens figure one, and multiply it by the tens, and annex the product of the units.

Rule Eleven.—To multiply fractional numbers, when the whole numbers are eqnal, and the

sum of the fractional parts make one, increase the whole number by one, and multiply by the whole number, and annex the product of the fractional parts.

RULE TWELVE. — Multiplying the unity term of any number divides the number; Dividing the unity term multiplies the number.

BUYING AND SELLING LUMBER.

RULE THIRTEEN. — Remove the decimal point three places to the left in any number of feet, and multiply by the price of one thousand feet in all examples.

BUYING AND SELLING BY THE HUNDRED.

RULE FOURTEEN. — Remove the decimal point two places to the left in the number of feet or number of pounds, and multiply by the price of one hundred in all examples.

BUYING AND SELLING BY THE TON.

RULE FIFTEEN. — Rule for all examples.

Remove the decimal point three places to the left, and multiply by one-half of the price per ton in all examples.

RULE SIXTEEN. — To find the number of short tons in any number of long tons. Multiply the expression, 1.12, by the number of long tons, and the result is the equivalent in short tons.

Rule Seventeen. — Buying and selling the long ton. Multiply the number of tons by the price of one ton. To find the price of odd pounds, remove the decimal point one place to the left in the price per ton, and divide by two, and you have the cost of 112 pounds; increase or diminish to find the cost of any number of pounds; thus for 14 pounds divide by 8, 28 pounds by 4, 56 pounds by 2 etc.

WEIGHING AND MEASURING GRAIN.

Rule Eighteen.—Find the number of cubic feet, remove the decimal point one place to the left and multiply by 8, and add 4½ bushels for each thousand.

Weigh one bushel and multiply by the number of bushels, and it is weighed. To find the number of gallons, multiply the number of bushels by 8.

Rule Nineteen. — To measure corn on the cob. Find the number of cubic feet in the bin, remove the decimal point one place to the left and multiply by 4½, and you have the number of bushels of shelled corn.

FOR MEASURING LAND.

Rule Twenty.—Remove the decimal point two places to the left in the number of rods, divide by 8 and multiply by 5, and you have the number of acrès.

Rule Twenty-one.— Remove the decimal point one place to the left in the number of square chains, in all examples.

RULE TWENTY-TWO.—Buying and selling grain by the quantity, to find the gain or loss.

Remove the decimal point two places to the left in the number of bushels and multiply by what it raises or falls per bushel.

RULE TWENTY-THREE.—Selling all articles bought by the dozen to make 20 per cent.

Remove the decimal point one place to the left in the price per dozen. Increase or diminish to reach all other per cents.

RULE TWENTY-FOUR. — For changing gold for currency.

Take 100 for the numerator, and the price of gold or currency, as the case may be, for the denominator, annex ciphers to the numerator and divide by the denominator, and you have the value of gold or currency. Or divide 10000 by the price of gold or currency.

RULE TWENTY-FIVE. — For calculating interest on any sum of money at any rate per cent.

Invert the rate per cent, and annex ciphers and prefix points, establishes the periods of time that it takes a dollar to earn a mill, cent, dime and dollar. Thus 6 per cent., 6 ds., 2 mo., 20 mo., 200 mo.

$7\frac{3}{10}$	"	5 ds., 50 ds., 500 ds., 5000 ds.,
$4\frac{1}{2}$	"	8ds., 80 ds., 800 ds., 8000 ds.
9	"	4 ds., 40 ds., 400 ds., 4000 ds.
8	"	4.5 ds., 45 ds., 15 mo., 150 mo.
12	"	3 ds., 1 mo., 10 mo., 100 mo.

$100	0	0	0.	00
523	4	6	8	75
14	6	8	7.	97
	6	9	8.	50

For the time it takes a dollar to earn a mill remove the decimal point three places to the left in any sum of money, a cent two places, a dime one place to the left, and when a dollar earns a dollar the note is the interest, and the decimal point remains unchanged. The same rule tells you the sum of money, at any rate per cent, it takes to earn a mill, a cent, a dime and a dollar in a day. Hence you can remove the decimal point in the number of days to find the interest, at any rate per cent and any period of time.

RULE TWENTY-SIX. — To find the square root of any number, divide it by the square of two and extract square root of quotient, and you have one-half the root of the number; divide it by the square of three, extract the square root of the quotient, and the result is one-third of the square root of the number, etc. Or divide any number by the square of any other number, and extract the square root of the quotient, and multiply the root found by the number that you square, and the result is the square root of the number. Or remove the decimal point two places to the left in any number, and find the square root of the quotient, and the result is one-tenth of the root; remove the point four places to the left, extract square root of the quotient, and you have one-hundreth part of the root, etc.

CUBE ROOT.

RULE TWENTY-SEVEN. — Divide any number by the cube of 2, or eight, extract cube root of the quotient, and the result is one-half of the cube root of the number. Divide any number by the cube of any other number, and extract the cube root of the quotient, and multiply the root found by the number that you cube and the result is the cube root of the number.

RULE TWENTY-EIGHT. — The sum of three sides of the complete cube always represents the trial divisor. To find each trial divisor, add what is shown as vacant in the engraving to the last true divisor. Each true divisor is found by adding to the trial divisor three times the surface of one side of each parrellelopiped, and one side of the small cube.

RULE TWENTY-NINE.— To measure stone.

Remove the decimal point two places to the left in the number of cubic feet and multiply by four and add to the result the $\frac{1}{100}$ part and you have the number of perches.

When in the form of the frustrum of a cone.

RULE.— Add the surface of both ends to the square root of the product of the surfaces of both ends, and multiply by one-third of the length.

RULE THIRTY. — A tree one hundred feet high blown over in a storm, the top resting on the ground forty feet from the root, and the butt on the stump, to find length of part broken off, and height of

stump. Rule for this and all similar problems. Square the distance from the root of the tree to where the top strikes, and divide the result by the length of the tree, and add the quotient to the length and divide by two, and you have the length of the part broken off.

RULE THIRTY-ONE.— To find the number of tons of iron it takes to build any number of miles of railroad.

Multiply the expression $\frac{11}{7}$ by the number of pounds one yard weighs, and that result by the number of miles, and you have the number of tons in all possible examples.

To find the least common multiple of two or more fractions.

RULE.—The greatest common divisor of the unity terms of the fractions is the unity term in the answer, and the least common multiple of the unit terms of the fractions is the unit term of the answer.

To find the greatest common divisor of two or more fractions.

RULE.— The least common multiple of the unity terms of the fractions is the unity term of the answer, and the greatest common divisor of the unit terms of the fractions is the unit term of the answer.

To find how much of an equal quantity of each of several articles at different prices can be bought or sold for a unit of money.

RULE.— Find the sum of the cost of the different articles, and *invert it*, and you have how much of an equal quantity can be had for a cent, a dime, or dollar as the case may be in all examples.

Example. — Three kinds of feed are worth $1.12½ $1.37½ and $1.50 per hundred pounds. How much of an equal quantity of each can be bought for $1 ?

Simply the sum of the cost of the different articles inverted, which is ¼, and $\frac{1}{4}\times 100$ is 25 pounds, which can be had for $1; for a dime 2.5; for a cent .25 of a pound.

At a teachers examination thirty-six failed to get a certificate, because they could not perform the following problem. The united yearly salaries of two teachers is 4400 francs. The first spends three-fourths of her yearly salary, and the second two-thirds of hers, they together have left 1310 francs. What was the yearly salary of each ?

Solution. — Let $\frac{1}{1}$ equal the firsts yearly salary $\frac{1'}{1}$ prime the seconds yearly salary.

Hence $\frac{1}{1}+\frac{1'}{1}=4400$ francs.

and $\frac{1}{4}+\frac{1'}{3}=1310$ "

Now $\frac{1}{4}+\frac{1'}{4}=1100$ "

and subtracting third equation from 2d we have $\frac{1'}{12}=210$ "

Hence $\frac{1'}{1}=2520$ the 2ds yearly salary.

and $\frac{1}{1}=1880$ the 1st " "

THE LAW OF TIME,

CONTAINING A BRIEF MENTAL RULE FOR EXAMINING THE DATE OF DEEDS, NOTES, RECEIPTS, MORTGAGES, AND RECTIFYING FAMILY RECORDS, WRITING LETTERS, READING HISTORY &C. &C.

The months, years and centuries have what we denominate *ratios*, because ratio means the quotient of one number divided by another — and in this rule the measuring unity is the number of days in a week, or 7, and is always the divisor and the dividend is the number of days past, or to come.

The student must observe, that the ratios of the months, years and centuries are simply fractional parts of a week, and may be called odd days, since all time past is an exact number of weeks or an exact number of weeks, and a fractional part of a week.

RULE FOR ALL TIME PAST AND FUTURE.

Unite the ratios of the century, the year, the month, and the date of the month in one sum, rejecting the sevens, the excess determines the day of the week.

When the excess is 1, it is the first day of the week or Sunday.

When the excess is 2 it is Monday,
" " " 3 " Tuesday,
" " " 4 " Wednesday,
" " " 5 " Thursday,
" " " 6 " Friday,
" " " 0 " Saturday.

14

Example 1.— The first book printed, that bears a date was published August 18th, 1477, by Wm. Caxton, England, a single copy of which has been recently sold for $10,000, and is now on exhibition in New York. — Required the day of its publication. —

Operation.— 2 is the ratio of the 15th century.
5 " " " year 1477.
5 " " " month August.
18 the date of the month.

$$7\overline{)30}$$
$$4-2$$

2 is the excess of sevens and is the second day of the week or Monday. Or unite the ratios by the sign of addition, thus:

$$\frac{2+5+5+18}{7} = 4\tfrac{2}{7}$$

rejecting the sevens we have $\frac{2}{7}$ of a week or the second day.

Example 2.— Required the day of the week of August 18th, 1877.

Statement. — $\frac{0+5+5+18}{7}$

rejecting the sevens we have 0 for the excess, hence it was Satuaday.

The United States Intellectual and Practical Lightning Calculator, published by the author, J. A. Henderson, A. M., Jan. 7, 1878. Required the day of the week.

Statement. — $\frac{0+6+3+7}{7}$

rejecting the sevens the excess is 2 or Monday.

Note 1. — Rejecting the sevens in the ratios to save adding you have an instant rule. A child can make the calculation for any date instantaneously.

Note 2. — The ratio of each month is found above its name.

3	6	6	2	4	0	2	5	1	3	6	1
Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.

Note 3.— To memorize the ratios of the months you know the name and arrangement of the months, hence associate the ratio of each month with its name.

Note 4. — The ratios of the months are established facts, and never change, only January and February. Leap year as it comes around causes their ratios to be 1 less, January 2 and February 5.

ARRANGEMENT OF THE RATIOS OF THE YEAR.

The year	the ratio is	The year	the ratio is
1	1	15	4
2	2	16	6
3	3	17	0
4	5	18	1
5	6	19	2
6	0	20	4
7	1	21	5
8	3	22	6
9	4	23	0
10	5	24	2
11	6	25	3
12	1	26	4
13	2	27	5
14	3	28	0

The year	29	the ratio is	1	The year	65	the ratio is	4
	30		2		66		5
	31		3		67		6
	32		5		68		1
	33		6		69		2
	34		0		70		3
	35		1		71		4
	36		3		72		6
	37		4		73		0
	38		5		74		1
	39		6		75		2
	40		1		76		4
	41		2		77		5
	42		3		78		6
	43		4		79		0
	44		6		80		2
	45		0		81		3
	46		1		82		4
	47		2		83		5
	48		4		84		0
	49		5		85		1
	50		6		86		2
	51		0		87		3
	52		2		88		5
	53		3		89		6
	54		4		90		0
	55		5		91		1
	56		0		92		3
	57		1		93		4
	58		2		94		5
	59		3		95		6
	60		5		96		1
	61		6		97		2
	62		0		98		3
	63		1		99		4
	64		3				

Note 1. — The fourth year you add one day, for it is leap year, hence its ratio is five.

Note 2. — the 6th becomes naught, because the leap year adds one day to 6, making 7, hence the excess or ratio becomes naught.

Note 3.— These ratios begin to repeat at the 7th year, and will continue to repeat by the same law.

TO FIND THE RATIO OF ANY YEAR.

RULE. — The excess of sevens in the last two figures of the year, and the fourth part of the last two figures, rejecting the fractional quotient when dividing by 4, is the ratio of any year.

Note 1.— Reject the fractional quotient when dividing by 4 because it is a fraction of a day and does not change the day of the week.

Note 2.—Since the ratio of one year is the excess of sevens in 365¼, or 1¼ days the ratio of any year is the excess of sevens in the year multiplied by 1¼. To multiply by 1¼ divide by 8 and call it tens.

ARRANGEMENT OF THE RATIOS OF THE CENTURIES.

16th	century	the	ratio is	1
15th	“	‘	“	2
14th	“	“	“	3
13th	“	“	“	4
12th	“	“	“	5
11th	“	“	“	6
10th	“	“	“	0
9th	“	“	“	1
8th	“	“	“	2
7th	“	“	“	3
6th	“	“	“	4
5th	“	“	“	5
4th	“	“	“	6
3d	“	“	“	0
2d	“	“	“	1
1st	“	“	“	2
17th	“	“	“	4
21st	“	“	“	4
25th	“	“	“	4
29th	“	“	“	4
20th	“	“	“	5
24th	“	“	“	5
28th	“	“	“	5
18th	“	“	“	2
22d	“	“	“	2
26th	“	“	“	2
30th	“	“	“	2
19th	“	“	“	0
23d	“	“	“	0
27th	“	“	“	0

You observe by the arrangemeut of the ratios you can memorize them in two minutes.

CERTIFICATES OF APPROVAL.

Page 903 of Henderson's Complete Home Instructor of the Science of Number.

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CERTIFICATES OF APPROVAL.

I have examined the new methods of calculating by Prof. J. A. Henderson, they are invaluable to business men, and will prove a light in science to all coming generations. A. J. WARNER,
Pres. Elmira Commercial College.

Henderson's methods are the finest known for lightning multiplication. PROF. D. R. FORD,
Female College, Elmira.

I have examined Prof. J. A. Henderson's new methods of calculation; they are remarkable for originality and of great practical value. His methods of calculating interest are peculiarly clear and comprehensive in their adaptation to all possible cases.
REV. DR. O. P. FITZGERALD.
Ex. State Superintendent, Cal.

Mr. J. A. Henderson has taught mathematics in Delhi Academy for a year. We consider him an excellent mathematical teacher. J. L. SAWYER,
Principal of Delhi Academy.

Delhi, Oct., 1862.

P. S.—J. A. H., taught analytical Trigonometry, University Algebra, Intellectual Arithmetic and English Grammar in Delhi Academy, New York.

John Alexander Henderson, A. M., attended Union College and graduated with me in class "64." He is an excellent scholar—among the first—and his character is above reproach. ELISHA CURTIS, A. M.
Principal of Sodus Academy.

I have known Prof. J. A. Henderson from earliest boyhood; his character has always been beyond reproach. As a mathematician he has scarcely an eqnal; as a teacher he has been eminently successful; as a phrenologist he is considered by many not a whit behind Fowler & Wells, N. Y. REV. A. G. KING,
of U. P. Church, N. Y., 1869.

ELMIRA, June 17, 1882.

J. A. HENDERSON, A. M., Buffalo, N. Y.:

Dear Sir—Having examined your methods of computation, I am compelled to acknowledge that they are original, simple and extraordinarily effective and expeditious in producing correct results. You certainly are deserving of much credit. Respectfully,

A. J. WARNER,

President Elmira Commercial College, N. Y.

TO WHOM IT MAY CONCERN.

I believe Prof. J. A. Henderson to be the best mathematician I have ever known. I heartily recommend his methods of calculation, and am confident that he will, in a great measure, revolutionize the old and tedious systems of mathematical computation.

F. E. WOOD,

President Williamsport Commercial College, Pa.

September 21, 1881.

I consider Prof. J. A. Henderson's unity and decimal method of calculating scientific in theory, clear in analysis, and ample in illustration. In reference to all business transactions which usually occur the rules are complete. The principles developed will solve all cases that are known, or likely to occur. The book is convenient and useful, and every person would do well to procure a copy and have it in possession, at all times, and under all circumstances.

JOSEPH W. WELTON,

Prof. of Mathematics in Grand Rapids Business College.

EVANSVILLE COMMERCIAL COLLEGE,
Evansville, Ind., Feb. 27, 1880.

PROF. J. A. HENDERSON, Nashville, Tenn.:

Dear Sir—We are using your Lightning Calculator in College and are highly pleased with it—wouldn't be be without it for any consideration. It has created the most intense excitement among our students, and an entire revolution in mathematics is the result. Figures are handled daily with almost incredible rapidity, and long, tedious, and laborious calculations of former days are changed, and have become a pleasure and pastime. Therefore we are compelled to acknowledge that words are inadequate to express our high opinion of your invaluable work. G. W. RANK, for RANK & WRIGHT.

GRATTAN, MICH., July 12th, 1879.

PROF. J. A. HENDERSON:

Dear Sir—Having written a treatise on the University Algebra, and being somewhat interested in brief and practical methods of mathematical calculations, my curiosity was very much startled at the lightning dispatch with which the most intricate and complicated propositions in mathematics can be solved by your Unity and Decimal method. This curiosity led me to a rigid examination of the work, which I find thorough in its treatment, accurate in its results, and highly pleasing in its simplicity. It is the only system of reasoning that answers a mathematical question in the same time it is asked, and the time is not far distant when it must necessarily be introduced into our common schools, as it is destined to revolutionize the mathematical world.

S. F. KENNEDY, Superintendent of Schools.

Prof. J. A. Henderson, of St. Louis, Mo., author of Henderson's Intellectual and Practical Lightning Calculator, was in town Friday and Saturday, and sold several copies of his excellent work. The editor of this paper, when fourteen years of age, was a member of Prof. Henderson's class in algebra, the Professor being at that time the best mathematician and teacher ever connected with the faculty of the Delaware Academy, Delhi, N. Y. Since that time the Professor has been a diligent worker, and his labor has been rewarded—he having sold over sixty thousand copies of his Lightning Calculator. No mathematical problem has been found too intricate for him to solve, and if he has any equals in this branch of study they are very few and far between. J. W. HINE, Editor Lowell Journal.

STILLWATER, MINN., June 24, 1879.

I have carefully examined J. A. Henderson's United States Unity and Decimal method, and do not hesitate to pronounce it the most valuable work of the kind for practical every-day use ever published. I have personally known Mr. Henderson since 1860 when he was Professor of Mathematics in Delaware Academy, at Delhi, N. Y., where he taught with great acceptance and success. I know him to be one of the best mathematicians of the age, as well as a thorough reliable gentleman in all the walks of life. I cheerfully recommend his book and himself to the favorable notice of the public. E. G. BUTTS, Postmaster.

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Address the Author:

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